Contract-based Discovery and Adaptation of Web Services

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9th International School on Formal Methods for the Design of Computer, Communication and Software Systems: Web Services

Contracts for Web services

We've got...

- behavioral descriptions of Web services (wsdl, wscl, ws-bpel, put your favorite technology here)
- repositories of Web services descriptions (uddi)

We'd like to...

- look for Web services with a given behavior
- see if it's safe to replace a service with another one
- if not, see whether we can *adapt* one service to safely replace another

We're gonna use...

- contracts = abstractions of Web services' behavior

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Finding Web services by contract

Compliance = client's satisfaction

$$\rho \dashv \sigma$$

Running a query with *compliance*

$$\mathcal{Q}(\rho) = \{\sigma \mid \rho \dashv \sigma\}$$

Running a query with *duality* $\rho^\perp$ and *subcontract* $\sigma \preceq \tau$

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The quest for $\preceq$

Desired properties of $\preceq$

- reduction of nondeterminism ($a \oplus b \preceq a$)
- extension of functionalities ($a \preceq a + b$)
- some permutation of messages ($a.c \preceq c.a$)

The problem

- *reduction* alone is too strict
- *extension* is unsafe
- *extension;reduction* is not transitive
- *permutation* is not allowed

The solution

- use (simple) orchestrators

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Summary

1. contracts
2. simple orchestrators
3. simple orchestrators with buffers
4. duality
5. recursive behaviors
6. orchestrator synthesis
7. related and ongoing work

Oh no! More ws-bpel!

```
<process>
  <sequence>
    <receive operation="Order" variable="Request"/>

    <flow>
      <invoke operation="Deposit" inputVariable="Request" outputVariable="Deposit"/>
      <invoke operation="Charge" inputVariable="Request" outputVariable="Charge"/>
    </flow>

    <switch>
      <case condition="getVariableData(Deposit) == true && getVariableData(Charge) == true)">
        <invoke operation="Ship" inputVariable="Request"/>
        <reply operation="Order" value="OK"/>
      </case>

      <case condition="getVariableData(Charge) == true)">
        <invoke operation="Refund" inputVariable="Request"/>
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What's in a contract

Actions

O	Order
D	Deposit
C	Charge
S	Ship
R	Refund

Traces

$$\{ODCSO, OCDSO, ODCRO, OCDRO, ODCO, OCDO\}$$

Branching points

- OD... and OC... is an *external choice*
- ...SO, ...RO, and ...O is an *internal choice*

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Basic theory of contracts

Contracts

Syntax

σ	$::=$		**contract**	α	$::=$		**action**
		0	(null)			a	(input)
	$\mid$	$\alpha.\sigma$	(action prefix)		$\mid$	$\bar{a}$	(output)
	$\mid$	$\sigma + \sigma$	(external choice)				
	$\mid$	$\sigma \oplus \sigma$	(internal choice)				

Example

$$\mathtt{O}.(\overline{\mathtt{D}}.\overline{\mathtt{C}}.\mathtt{D}.\mathtt{C}.(\overline{\mathtt{S}}.\overline{\mathtt{O}} \oplus \overline{\mathtt{R}}.\overline{\mathtt{O}} \oplus \overline{\mathtt{O}}) + \overline{\mathtt{C}}.\overline{\mathtt{D}}.\mathtt{D}.\mathtt{C}.(\overline{\mathtt{S}}.\overline{\mathtt{O}} \oplus \overline{\mathtt{R}}.\overline{\mathtt{O}} \oplus \overline{\mathtt{O}}))$$

Operational semantics

$$\alpha.\sigma \xrightarrow{\alpha} \sigma \qquad \sigma \oplus \tau \longrightarrow \sigma \qquad \frac{\sigma \xrightarrow{\alpha} \sigma'}{\sigma + \tau \xrightarrow{\alpha} \sigma'} \qquad \frac{\sigma \longrightarrow \sigma'}{\sigma + \tau \longrightarrow \sigma' + \tau}$$

Compliance, formally

Systems

$$\rho \parallel \sigma$$

System transitions

$$\frac{\rho \longrightarrow \rho'}{\rho \parallel \sigma \longrightarrow \rho' \parallel \sigma} \qquad \frac{\sigma \longrightarrow \sigma'}{\rho \parallel \sigma \longrightarrow \rho \parallel \sigma'} \qquad \frac{\rho \xrightarrow{\overline{\alpha}} \rho' \qquad \sigma \xrightarrow{\alpha} \sigma'}{\rho \parallel \sigma \longrightarrow \rho' \parallel \sigma'}$$

Compliance

$$\rho \dashv \sigma \quad \stackrel{\text{def}}{\iff} \quad \rho \parallel \sigma \Longrightarrow \rho' \parallel \sigma' \not\longrightarrow \text{ implies } \rho' \xrightarrow{\mathsf{e}}$$

Compliance, examples

$$\bar{a}.\mathsf{e} \dashv ? \; a$$

$$\bar{a}.\mathsf{e} \oplus \bar{b}.\mathsf{e} \dashv a + b$$

$$\mathsf{e} \dashv \sigma$$

$$\bar{a}.\mathsf{e} \oplus \bar{b}.\mathsf{e} \dashv a \oplus b$$

$$\bar{a}.\mathsf{e} \dashv a \oplus 0$$

$$0 \dashv \sigma$$

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$$\bar{a}.\mathsf{e} \dashv a \quad ☺$$

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Subcontract, formally

Set-theoretic interpretation of contracts

$$[\![\sigma]\!]^{\mathsf{s}} \stackrel{\text{def}}{=} \{\rho \mid \rho \dashv \sigma\}$$

Subcontract

$$\sigma \sqsubseteq \tau \stackrel{\text{def}}{\iff} [\![\sigma]\!]^{\mathsf{s}} \subseteq [\![\tau]\!]^{\mathsf{s}}$$

$$\simeq \; = \; \sqsubseteq \cap \sqsupseteq$$

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Subcontract, (counter)examples

$\overline{a}.e$	$a + b$	$\not\sqsubseteq$	$a \oplus b$
$\overline{a}.e$	a	$\not\sqsubseteq$	0
$e + a$	0	$\not\sqsubseteq$	$\overline{a}$

Reduction of nondeterminism

$$\sigma \oplus \tau \sqsubseteq \sigma$$

Subcontract, (counter)examples

$\overline{a}.\mathsf{e}$		$a + b \not\sqsubseteq a \oplus b$
$\overline{a}.\mathsf{e}$		$a \not\sqsubseteq 0$
$\mathsf{e} + a$		$0 \not\sqsubseteq \overline{a}$

Reduction of nondeterminism

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Properties of strong subcontract

Internal choice = intersection

$$[\![\sigma \oplus \tau]\!]^{\mathsf{s}} = [\![\sigma]\!]^{\mathsf{s}} \cap [\![\tau]\!]^{\mathsf{s}}$$

External choice $\neq$ union

- there are clients in $[\![a + b]\!]^{\mathsf{s}}$ that are not in $[\![a]\!]^{\mathsf{s}} \cup [\![b]\!]^{\mathsf{s}}$:

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- sometimes $+$ is $\oplus$ in disguise:

$$\alpha.\sigma + \alpha.\tau \simeq \alpha.(\sigma \oplus \tau)$$

- interferences:

$$\overline{a}.\mathsf{e} + \overline{b} \in [\![a]\!]^{\mathsf{s}} \qquad \overline{a}.\mathsf{e} + \overline{b} \notin [\![a + b]\!]^{\mathsf{s}}$$

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External choice ≠ union

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Properties of strong subcontract

Proposition

$\sqsubseteq$ *is a precongruence*

$$\sigma \sqsubseteq \tau \qquad \Rightarrow \qquad \begin{cases} \alpha.\sigma \sqsubseteq \alpha.\tau \\ \sigma \oplus \sigma' \sqsubseteq \tau \oplus \sigma' \\ \sigma + \sigma' \sqsubseteq \tau + \sigma' \end{cases}$$

+ nice axiomatization
+ can be used for safe replacement of parts of services

Strong subcontract: axioms

$$
\begin{array}{lrcl}
\text{(e1)} & \sigma + \sigma & = & \sigma \\
\text{(e2)} & \sigma + \tau & = & \tau + \sigma \\
\text{(e3)} & \sigma + (\sigma' + \sigma'') & = & (\sigma + \sigma') + \sigma'' \\
\text{(e4)} & \sigma + 0 & = & \sigma \\
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\text{(i1)} & \sigma \oplus \sigma & = & \sigma \\
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\text{(d3)} & \alpha.\sigma + \alpha.\tau & = & \alpha.(\sigma \oplus \tau) \\
\text{(d4)} & \alpha.\sigma \oplus \alpha.\tau & = & \alpha.(\sigma \oplus \tau) \\
\\
\text{(red)} & \sigma \oplus \tau & \leq & \sigma
\end{array}
$$

Orchestrators

Limitations of $\sqsubseteq$

$\sqsubseteq$ does not support extensions. . .

$$a \not\sqsubseteq a + b$$

. . . because extra actions may cause interferences

$$\overline{a}.\mathsf{e} + \overline{b} \dashv a \qquad\qquad \overline{a}.\mathsf{e} + \overline{b} \not\dashv a + b$$

Why not all failures are equal

Failure due to client nondeterminism

$$\overline{a}.\mathsf{e} \oplus \overline{b}.\mathsf{e} \parallel a \longrightarrow \overline{b}.\mathsf{e} \parallel a$$

Failure due to service nondeterminism

$$\overline{a}.\mathsf{e} \parallel a \oplus b \longrightarrow \overline{a}.\mathsf{e} \parallel a$$

Failure due to "system" nondeterminism

$$\overline{a}.\mathsf{e} + \overline{b}.c.\mathsf{e} \parallel a + b.\overline{d} \longrightarrow c.\mathsf{e} \parallel \overline{d}$$

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Idea: orchestrated interaction

client	⇆	orchestrator	⇆	service
$\overline{a}$	$\rightarrow$	$\langle a, \overline{a} \rangle$	$\rightarrow$	a
a	$\leftarrow$	$\langle \overline{a}, a \rangle$	$\leftarrow$	$\overline{a}$
$\overline{\alpha}$				α
$\overline{\alpha}$		$\langle \alpha, \overline{\alpha} \rangle$		

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a	$\leftarrow$	$\langle \overline{a}, a \rangle$	$\leftarrow$	$\overline{a}$
$\overline{\alpha}$				α
$\overline{\alpha}$		$\langle \alpha, \overline{\alpha} \rangle$		

Synchronous orchestrators

f	$::=$		**orchestrator**
		0	(null)
	$\mid$	$\mu.f$	(action prefix)
	$\mid$	$f \vee f$	(disjunction)

μ	$::=$		**orchestration action**
		$\langle a, \overline{a} \rangle$	(input/output)
	$\mid$	$\langle \overline{a}, a \rangle$	(output/input)

Orchestrator semantics

Orchestrator transitions

$$\mu.f \xrightarrow{\mu} f \qquad \frac{f \xrightarrow{\mu} f'}{f \vee g \xrightarrow{\mu} f'}$$

Trace semantics for orchestrators

$$[\![f]\!] \stackrel{\text{def}}{=} \{\mu_1 \cdots \mu_n \mid \exists g : f \xrightarrow{\mu_1} \cdots \xrightarrow{\mu_n} g\}$$

Orchestrated compliance, formally

Orchestrated systems

$$\rho \parallel_f \sigma$$

Orchestrated system transitions

$$\frac{\rho \longrightarrow \rho'}{\rho \parallel_f \sigma \longrightarrow \rho' \parallel_f \sigma} \qquad \frac{\sigma \longrightarrow \sigma'}{\rho \parallel_f \sigma \longrightarrow \rho \parallel_f \sigma'}$$

$$\frac{\rho \xrightarrow{\overline{\alpha}} \rho' \quad f \xrightarrow{\langle \alpha, \overline{\alpha} \rangle} f' \quad \sigma \xrightarrow{\alpha} \sigma'}{\rho \parallel_f \sigma \longrightarrow \rho' \parallel_{f'} \sigma'}$$

Weak compliance

$$f : \rho \dashv\!\!\!\vdots\; \sigma \stackrel{\text{def}}{\iff} \rho \parallel_f \sigma \Longrightarrow \rho' \parallel_{f'} \sigma' \not\longrightarrow \text{ implies } \rho' \xrightarrow{\mathsf{e}}$$

Weak compliance, examples

$$\langle a, \overline{a} \rangle \quad : \quad \overline{a}.\mathsf{e} \ \dashv\!\!\!\mid\!\mid \ ? \ \ a + b$$

$$\langle a, \overline{a} \rangle \vee \langle b, \overline{b} \rangle \quad : \quad \overline{a}.\mathsf{e} \ \dashv\!\!\!\mid\!\mid \ a + b$$

$$\langle a, \overline{a} \rangle \vee \langle b, \overline{b} \rangle \quad : \quad \overline{a}.\mathsf{e} \oplus \overline{b}.\mathsf{e} \ \dashv\!\!\!\mid\!\mid \ a + b$$

$$0 \quad : \quad \mathsf{e} + a \ \dashv\!\!\!\mid\!\mid \ \overline{a}$$

$$f \quad : \quad \overline{a}.\mathsf{e} \oplus \overline{b}.\mathsf{e} \ \dashv\!\!\!\mid\!\mid \ a \oplus b$$

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Weak subcontract, formally

Set-theoretic interpretation of contracts

$$[\![\sigma]\!]^{\mathsf{w}} \stackrel{\text{def}}{=} \{\rho \mid \exists f : f : \rho \dashv\!\!\! \mid\mid \sigma\}$$

Weak subcontract

$$\sigma \preceq \tau \stackrel{\text{def}}{\iff} [\![\sigma]\!]^{\mathsf{s}} \subseteq [\![\tau]\!]^{\mathsf{w}}$$

A few doubts. . .

- is $\preceq$ the subcontract relation we're looking for?
- is $\preceq$ a preorder?

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Universal orchestrators

$$\sigma \preceq \tau \iff \text{for every } \rho, \rho \dashv \sigma \text{ implies } f : \rho \dashv\!\!\mid \tau \text{ for some } f$$

Universal orchestrator

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f is the *universal orchestrator* for $\sigma \preceq \tau$

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Orchestrators as morphisms

Orchestrator application $f(\sigma)$

f	σ	$f(\sigma)$
$\langle a, \overline{a} \rangle \vee \langle b, \overline{b} \rangle$	$a + b$	$a + b$
$\langle a, \overline{a} \rangle \vee \langle b, \overline{b} \rangle$	$a \oplus b$	$a \oplus b$
$\langle a, \overline{a} \rangle$	$a + b$	a
$\langle a, \overline{a} \rangle$	$a \oplus b$	$a \oplus 0$
0	σ	0

Theorem

$f : \rho \dashv\!| \, \sigma$ *if and only if* $\rho \dashv f(\sigma)$

Corollary

$f : \sigma \preceq \tau$ *if and only if* $\sigma \sqsubseteq f(\tau)$

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Weak subcontract, examples

Reduction of nondeterminism ($\preceq$ embeds $\sqsubseteq$)

$$\langle a, \overline{a} \rangle \vee \langle b, \overline{b} \rangle : a \oplus b \preceq a$$
$$\Updownarrow$$
$$a \oplus b \sqsubseteq (\langle a, \overline{a} \rangle \vee \langle b, \overline{b} \rangle)(a) = a$$

Width extension

$$\langle a, \overline{a} \rangle : a \preceq a + b$$
$$\Updownarrow$$
$$a \sqsubseteq \langle a, \overline{a} \rangle (a + b) = a$$

Depth extension

$$0 : 0 \preceq \sigma$$
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$$0 \sqsubseteq 0(\sigma) = 0$$

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On transitivity of $\preceq$

Is $\preceq$ transitive?

$$f : \sigma \preceq \sigma' \qquad g : \sigma' \preceq \tau \qquad \stackrel{?}{\Longrightarrow} \qquad h : \sigma \preceq \tau$$

$$\begin{array}{rcrcl} \langle a, \overline{a} \rangle \vee \langle b, \overline{b} \rangle . \langle c, \overline{c} \rangle & : & a \oplus b.c & \preceq & a \\ \langle a, \overline{a} \rangle & : & a & \preceq & a + b.d \\ ??? & : & a \oplus b.c & \preceq & a + b.d \end{array}$$

Proposition (Orchestrator application is monotone)

$\sigma \sqsubseteq \tau$ *implies* $f(\sigma) \sqsubseteq f(\tau)$

$$\sigma \sqsubseteq f(\sigma') \qquad \sigma' \sqsubseteq g(\tau) \qquad \Longrightarrow \qquad \sigma \sqsubseteq f(g(\tau))$$

$\preceq$ is transitive if $f \circ g$ is an orchestrator

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Orchestrator composition and transitivity of $\preceq$

Orchestrator composition

$$f \wedge g$$

- $f \wedge g$ permits the traces permitted by f and by g
- $f \wedge g$ forbids the traces forbidden by either f or by g

$$[\![f \wedge g]\!] \stackrel{\text{def}}{=} [\![f]\!] \cap [\![g]\!]$$

Proposition

$f(g(\sigma)) \simeq (f \wedge g)(\sigma)$

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Towards a deduction system for $\preceq$

Problem: $\preceq$ is not a precongruence w.r.t. $+$

$$\begin{array}{ccccccc} & & 0 & \preceq & & & a \\ a.b & & & \preceq & a.b & & \\ a.b & + & 0 & \not\preceq & a.b & + & a \simeq a.(b \oplus 0) \end{array}$$

Proposition (distributivity of orchestration application)

1. $f(\sigma) + f(\tau) \simeq f(\sigma + \tau)$
2. $f(\sigma) \oplus f(\tau) \simeq f(\sigma \oplus \tau)$

Corollary

$f : \sigma_1 \preceq \tau_1$ *and* $f : \sigma_2 \preceq \tau_2$ *implies* $f : \sigma_1 + \sigma_2 \preceq \tau_1 + \tau_2$

- precongruence is granted if the orchestrator is oblivious of the particular branch taken

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Deduction system for $\preceq$

(red) $$I(\sigma) : \sigma \oplus \tau \leq \sigma$$

(prefix) $$\frac{f : \sigma \leq \tau}{\langle \alpha, \overline{\alpha} \rangle . f : \alpha.\sigma \leq \alpha.\tau}$$

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The deduction system is *sound* and *complete*

$$f : \sigma \preceq \tau \iff f : \sigma \leq \tau$$

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Interpretations of orchestrators

As **mediators**

$$\rho \parallel_f \sigma$$

As **morphisms/behavioral coercions**

$$f : \sigma \preceq \tau \iff \sigma \sqsubseteq f(\tau) \qquad f : \tau \to \sigma$$

As **assumptions** on the environment

$$\langle a, \overline{a} \rangle : a \preceq a + b$$

- it is safe to replace a with $a + b$ if nobody ever tries to perform $\overline{b}$

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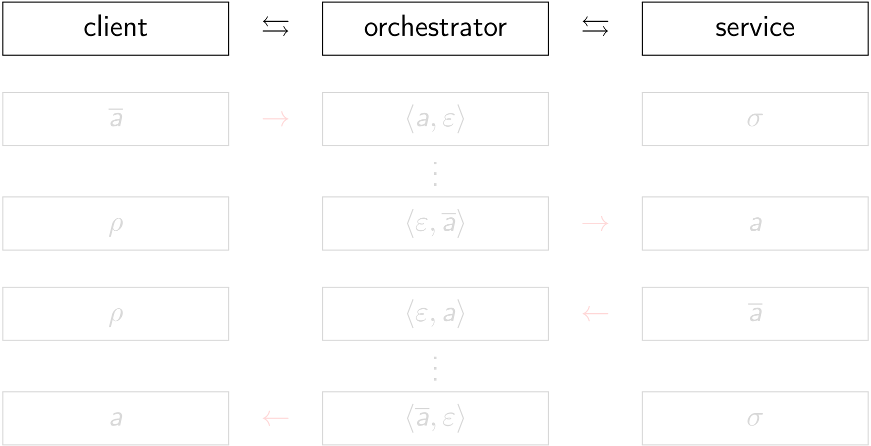
Buffered orchestrators

Buffered orchestrators



Buffered orchestrators

client	$\leftrightarrows$	orchestrator	$\leftrightarrows$	service
$\overline{a}$	$\rightarrow$	$\langle a, \varepsilon \rangle$		σ
		$\vdots$		
ρ		$\langle \varepsilon, \overline{a} \rangle$	$\rightarrow$	a
ρ		$\langle \varepsilon, a \rangle$	$\leftarrow$	$\overline{a}$
		$\vdots$		
a	$\leftarrow$	$\langle \overline{a}, \varepsilon \rangle$		σ

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client	$\leftrightarrows$	orchestrator	$\leftrightarrows$	service
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		$\vdots$		
a	$\leftarrow$	$\langle \overline{a}, \varepsilon \rangle$		σ

Syntax of buffered orchestrators

$$
\begin{array}{llll}
f & ::= & & \textbf{orchestrator} \\
 & & 0 & \text{(null)} \\
 & | & \mu.f & \text{(action prefix)} \\
 & | & f \vee f & \text{(disjunction)}
\end{array}
$$

$$
\begin{array}{llll}
\mu & ::= & & \textbf{orchestration action} \\
 & & \langle a, \overline{a} \rangle & \text{(input/output)} \\
 & | & \langle \overline{a}, a \rangle & \text{(output/input)} \\
 & | & \langle \alpha, \varepsilon \rangle & \text{(action/–)} \\
 & | & \langle \varepsilon, \alpha \rangle & \text{(–/action)}
\end{array}
$$

Valid k-orchestrators

Not every orchestrator makes sense

orchestrator	valid	rank
$\langle \varepsilon, a \rangle . \langle \overline{a}, \varepsilon \rangle$		
$\langle a, \varepsilon \rangle . \langle a, \varepsilon \rangle$		
$\langle \overline{a}, \varepsilon \rangle$		
$\langle \varepsilon, \overline{a} \rangle$		
$\langle a, \varepsilon \rangle . \langle \overline{a}, \varepsilon \rangle$		

Valid k-orchestrators are...

- directional
- finite-state
- fair

Valid k-orchestrators

Not every orchestrator makes sense

orchestrator	valid	rank
$\langle \varepsilon, a \rangle . \langle \overline{a}, \varepsilon \rangle$	☺	≥ 1
$\langle a, \varepsilon \rangle . \langle a, \varepsilon \rangle$		
$\langle \overline{a}, \varepsilon \rangle$		
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$\langle \overline{a}, \varepsilon \rangle$	☹	
$\langle \varepsilon, \overline{a} \rangle$	☹	
$\langle a, \varepsilon \rangle . \langle \overline{a}, \varepsilon \rangle$	☹	

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Weak k-compliance, formally

Orchestrated systems

$$\rho \parallel_f \sigma$$

Orchestrated system transitions

$$\cdots$$

$$\frac{\rho \xrightarrow{\overline{\alpha}} \rho' \quad f \xrightarrow{\langle \alpha, \varepsilon \rangle} f'}{\rho \parallel_f \sigma \longrightarrow \rho' \parallel_{f'} \sigma} \qquad \frac{f \xrightarrow{\langle \varepsilon, \overline{\alpha} \rangle} f' \quad \sigma \xrightarrow{\alpha} \sigma'}{\rho \parallel_f \sigma \longrightarrow \rho \parallel_{f'} \sigma'}$$

Weak k-compliance

$$f : \rho \dashv\!\mid_k \sigma \quad \overset{\text{def}}{\iff} \quad \rho \parallel_f \sigma \Longrightarrow \rho' \parallel_{f'} \sigma' \not\longrightarrow \text{ implies } \rho' \xrightarrow{\mathsf{e}}$$

Weak k-compliance, an example

$$\langle a, \varepsilon\rangle.\langle b, \varepsilon\rangle.\langle \varepsilon, b\rangle.\langle \varepsilon, a\rangle.\langle \overline{c}, c\rangle : \overline{a}.\overline{b}.c.\mathsf{e} \dashv\!\!\shortmid_1 b.a.\overline{c}$$

Weak k-subcontract, formally

Set-theoretic interpretation of contracts

$$[\![\sigma]\!]^{\mathsf{w}}_k \stackrel{\text{def}}{=} \{\rho \mid \exists f : f : \rho \dashv\!\!\vert_k \sigma\}$$

Weak subcontract

$$\sigma \preceq \tau \stackrel{\text{def}}{\iff} [\![\sigma]\!]^{\mathsf{s}} \subseteq [\![\tau]\!]^{\mathsf{w}}_k$$

Proposition (existence of universal orchestrator)

$\sigma \preceq_k \tau$ if and only if $f : \sigma \preceq_k \tau$ for some k-orchestrator f

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Orchestrators as morphisms

Orchestrator application $f(\sigma)$

f	σ	$f(\sigma)$
$\langle a, \varepsilon\rangle.\langle b, \overline{b}\rangle$	b	$a.b$
$\langle \varepsilon, \overline{a}\rangle.\langle b, \overline{b}\rangle$	$a.b$	b
$\langle a, \varepsilon\rangle.\langle b, \varepsilon\rangle.\langle \varepsilon, \overline{b}\rangle.\langle \varepsilon, \overline{a}\rangle$	$b.a$	$a.b$

Theorem

$f : \rho \dashv\!|_k \sigma$ *if and only if* $\rho \dashv f(\sigma)$

Corollary

$f : \sigma \preceq_k \tau$ *if and only if* $\sigma \sqsubseteq f(\tau)$

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$f : \sigma \preceq_k \tau$ *if and only if* $\sigma \sqsubseteq f(\tau)$

Orchestrator composition and transitivity of $\preceq_k$

We've got a problem

$$\begin{aligned}
\sigma &\stackrel{\text{def}}{=} \overline{b} + \overline{d} \\
f &\stackrel{\text{def}}{=} \langle a, \varepsilon\rangle.\langle c, \varepsilon\rangle.(\langle \varepsilon, \overline{a}\rangle.\langle \overline{b}, b\rangle \vee \langle \varepsilon, \overline{c}\rangle.\langle \overline{d}, d\rangle) \\
g &\stackrel{\text{def}}{=} \langle a, \varepsilon\rangle.\langle \overline{b}, b\rangle \vee \langle c, \varepsilon\rangle.\langle \overline{d}, d\rangle
\end{aligned}$$

$$f(g(\sigma)) \simeq f(a.\overline{b} + c.\overline{d}) \simeq a.c.(\overline{b} \oplus \overline{d})$$

No single orchestrator can turn $\overline{b} + \overline{d}$ into $a.c.(\overline{b} \oplus \overline{d})$

Idea

Find an orchestrator $f \cdot g$ such that $f(g(\sigma)) \sqsubseteq (f \cdot g)(\sigma)$

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Idea

Find an orchestrator $f \cdot g$ such that $f(g(\sigma)) \sqsubseteq (f \cdot g)(\sigma)$

Orchestrator composition, formally

$$f \cdot g \stackrel{\text{def}}{=} \bigvee_{f \xrightarrow{\langle \alpha, \varepsilon \rangle} f'} \langle \alpha, \varepsilon \rangle.(f' \cdot g) \vee \bigvee_{g \xrightarrow{\langle \varepsilon, \overline{\alpha} \rangle} g'} \langle \varepsilon, \overline{\alpha} \rangle.(f \cdot g')$$
$$\vee \bigvee_{f \xrightarrow{\langle \varphi, \overline{\alpha} \rangle} f', g \xrightarrow{\langle \alpha, \varphi' \rangle} g', \varphi\varphi' \neq \varepsilon} \langle \varphi, \varphi' \rangle.(f' \cdot g')$$
$$\vee \bigvee_{f \xrightarrow{\langle \varepsilon, \overline{\alpha} \rangle} f', g \xrightarrow{\langle \alpha, \varepsilon \rangle} g'} (f' \cdot g')$$

Theorem

$f(g(\sigma)) \sqsubseteq (f \cdot g)(\sigma)$

Deduction system for $\preceq_k$

No complete deduction system for $\preceq_k$ is known

(swap-inputs)	(swap-outputs)	(postpone-input)
$a.b.\sigma = b.a.\sigma$	$\overline{a}.\overline{b}.\sigma = \overline{b}.\overline{a}.\sigma$	$a.\overline{b}.\sigma \leq \overline{b}.a.\sigma$

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Duality

Duality

$$\rho^{\perp}$$

the $\preceq$-smallest service that satisfies ρ

Duality, examples

ρ	$\rho^{\perp}$
$a.\mathsf{e}$	
$a.\mathsf{e} \oplus b.\mathsf{e}$	
$a.\mathsf{e} + b.\mathsf{e}$	
e	
$a.\overline{b}.\mathsf{e} + a.\overline{c}.\mathsf{e}$	
0	
$a.\mathsf{e} \oplus 0$	
$a.0$	
$\dots$	

Definition (viable contract)

ρ is *viable* if there exists σ such that $\rho \dashv \sigma$

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$a.\overline{b}.\mathsf{e} + a.\overline{c}.\mathsf{e}$	$\overline{a}.(b + c)$
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$a.\overline{b}.\mathsf{e} + a.\overline{c}.\mathsf{e}$	$\overline{a}.(b + c)$
0	☹
$a.\mathsf{e} \oplus 0$	☹
$a.0$	☹
$\dots$	

Definition (viable contract)

ρ is *viable* if there exists σ such that $\rho \dashv \sigma$

Duality, formally

$$\rho^{\perp} \stackrel{\text{def}}{=} \sum_{\rho \Downarrow \mathsf{r}, \mathsf{e} \notin \mathsf{r}} \bigoplus_{\alpha \in \mathsf{r}, \mathsf{viable}(\rho(\alpha))} \overline{\alpha}.\rho(\alpha)^{\perp}$$

Theorem

Let ρ be a viable client contract. Then

1. $\rho \dashv \rho^{\perp}$
2. $\rho \dashv \sigma$ *implies* $\rho^{\perp} \preceq_0 \sigma$

Recursive behaviors

Finite syntax for finite behaviors

σ	$::=$		**contract**
		0	(null)
	$\mid$	$\alpha.\sigma$	(action prefix)
	$\mid$	$\sigma + \sigma$	(external choice)
	$\mid$	$\sigma \oplus \sigma$	(internal choice)

α	$::=$		**action**
		a	(input)
	$\mid$	$\overline{a}$	(output)

What about recursive behavior?

Describing recursive behavior

Many different ways. . .

$$\text{rec } X.a.X \oplus b.0$$

$$X = a.X \oplus b.0$$

$$\sigma^*$$

. . . **but it's just syntax!**

Infinite syntax for infinite behaviors

σ ::=		**contract**	
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α ::=		**action**
	a	(input)
$\mid$	$\overline{a}$	(output)

Definition

The set of contracts is the set of *possibly infinite trees* generated by the grammar above such that

1. they have *finitely many different* subtrees
2. every *infinite branch* has *infinitely many prefixes*

Every *finite* contracts satisfies these conditions

Examples

$$\begin{aligned} X &= a.X \\ &= a.a.X \\ &= \cdots \end{aligned}$$ ☺

$$\begin{aligned} X &= a.X \oplus b.0 \\ &= a.(a.X \oplus b.0) \oplus b.0 \\ &= a.(a.(a.X \oplus b.0) \oplus b.0) \oplus b.0 \\ &= a.(a.(a.(a.X \oplus b.0) \oplus b.0) \oplus b.0) \oplus b.0 \\ &= \cdots \end{aligned}$$ ☺

$$X = X + X$$ ☹

$$X = X \oplus X$$ ☹

$$X = X$$ ☹

Recursion: summary

- all the results stated previously still hold (coinduction)
- use your own preferred syntax (but beware of Kleene $*$)

Why does it work?

Proposition

Let $D(\sigma) \stackrel{\text{def}}{=} \{\sigma' \mid \sigma \stackrel{\varphi}{\Longrightarrow} \sigma'\}$. *Then* $D(\sigma)$ *is finite for every* σ

Algorithm

Towards an algorithm for deciding $\preceq_k$

Problem: the orchestrator is not necessarily unique

$$
\begin{array}{rcrcl}
0 & : & a \oplus b \oplus 0 & \preceq & a + b \\
\langle a, \overline{a} \rangle & : & a \oplus b \oplus 0 & \preceq & a + b \\
\langle b, \overline{b} \rangle & : & a \oplus b \oplus 0 & \preceq & a + b \\
\langle a, \overline{a} \rangle \vee \langle b, \overline{b} \rangle & : & a \oplus b \oplus 0 & \preceq & a + b
\end{array}
$$

$f \leqslant g$: g is better (more permissive) than f

$$f \leqslant g \overset{\text{def}}{\iff} [\![f]\!] \subseteq [\![g]\!]$$

Idea

- synthesize the best orchestrator
- the best orchestrator is unique

Towards an algorithm for deciding $\preceq_k$

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$$\begin{array}{rcccc} 0 & : & a \oplus b \oplus 0 & \preceq & a + b \\ \langle a, \overline{a} \rangle & : & a \oplus b \oplus 0 & \preceq & a + b \\ \langle b, \overline{b} \rangle & : & a \oplus b \oplus 0 & \preceq & a + b \\ \langle a, \overline{a} \rangle \vee \langle b, \overline{b} \rangle & : & a \oplus b \oplus 0 & \preceq & a + b \end{array}$$

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- the best orchestrator is unique

Deciding $\preceq_k$

$$\frac{\begin{array}{c} \mathsf{a}_r = \{\langle \varphi, \overline{\varphi}' \rangle \mid \sigma \stackrel{\varphi}{\Longrightarrow}, \tau \stackrel{\varphi'}{\Longrightarrow}, \mathbb{B} \vdash_k \langle \varphi, \overline{\varphi}' \rangle\} \\ \mathsf{a} = \{\langle \varphi, \overline{\varphi}' \rangle \in \mathsf{a}_r \mid \mathbb{B}\langle \varphi, \overline{\varphi}' \rangle \vdash_k f_{\langle \varphi, \overline{\varphi}' \rangle} : \sigma(\varphi) \trianglelefteq \tau(\varphi')\} \\ \tau \Downarrow \mathsf{s} \Rightarrow (\exists \mathsf{r} : \sigma \Downarrow \mathsf{r} \wedge \mathsf{r} \subseteq \mathsf{a} \circ \mathsf{s}) \vee (\emptyset \bullet \mathsf{a}) \cap \overline{\mathsf{s}} \neq \emptyset \end{array}}{\mathbb{B} \vdash_k \bigvee_{\mu \in \mathsf{a}} \mu.f_\mu : \sigma \trianglelefteq \tau}$$

Theorem

The following properties hold:

1. *(termination) the algorithm always terminates*
2. *(correctness)* $f : \sigma \trianglelefteq_k \tau$ *implies that* f *has rank* k *and* $f : \sigma \preceq_k \tau$
3. *(completeness)* $f : \sigma \preceq_k \tau$ *implies* $g : \sigma \trianglelefteq_k \tau$ *for some* $g \geqslant f$

An example from Wil's lecture (1/3)

$$\sigma \stackrel{\text{def}}{=} \mathit{order}.(\mathit{money} + \overline{\mathit{food}}.\mathit{money})$$

$$\rho_1 \stackrel{\text{def}}{=} \overline{\mathit{order}}.\mathit{food}.\overline{\mathit{money}}.\mathsf{e}$$

$$\rho_1^\perp = \mathit{order}.\overline{\mathit{food}}.\mathit{money}$$

$$f_1 = \langle \mathit{order}, \overline{\mathit{order}} \rangle.\langle \overline{\mathit{food}}, \mathit{food} \rangle.\langle \mathit{money}, \overline{\mathit{money}} \rangle$$

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An example from Wil's lecture (2/3)

$$\sigma \stackrel{\text{def}}{=} \mathit{order}.(\mathit{money} + \overline{\mathit{food}}.\mathit{money})$$

$$\rho_2 \stackrel{\text{def}}{=} \overline{\mathit{order}}.(\mathit{food}.\overline{\mathit{money}}.\mathsf{e} + \overline{\mathit{money}}.\mathit{food}.\mathsf{e})$$

$$\rho_2^{\perp} = \mathit{order}.(\overline{\mathit{food}}.\mathit{money} \oplus \mathit{money}.\overline{\mathit{food}})$$

$$f_2 = \langle \mathit{order}, \overline{\mathit{order}} \rangle.\langle \overline{\mathit{food}}, \mathit{food} \rangle.\langle \mathit{money}, \overline{\mathit{money}} \rangle$$

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An example from Wil's lecture (3/3)

$$\sigma \stackrel{\text{def}}{=} \mathit{order}.(\mathit{money} + \overline{\mathit{food}}.\mathit{money})$$

$$\rho_3 \stackrel{\text{def}}{=} \overline{\mathit{order}}.\overline{\mathit{money}}.\mathit{food}.\mathsf{e}$$

$$\rho_3^{\perp} = \mathit{order}.\mathit{money}.\overline{\mathit{food}}$$

$$f_3 = \langle \mathit{order}, \overline{\mathit{order}} \rangle.\langle \mathit{money}, \varepsilon \rangle.\langle \overline{\mathit{food}}, \mathit{food} \rangle.\langle \varepsilon, \overline{\mathit{money}} \rangle$$

An example from Wil's lecture (3/3)

$$\sigma \stackrel{\text{def}}{=} \mathit{order}.(\mathit{money} + \overline{\mathit{food}}.\mathit{money})$$

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Conclusions

Wrap-up

Subcontract relation

- tool for *searching* and *reasoning about* services by their contracts (= behavioral types)
- $\preceq$ combines reduction, extension, and permutation into a single preorder
- $\preceq$ gives safe substitution of services modulo orchestration
- $\preceq$ is decidable

(Simple) orchestrators

- have nice properties (universality, compositionality)
- can be automatically synthesized

Contracts vs session types

What is being typed

- contract = type of a process
- session type = type of a channel
- session type = type of a process projected on a channel

Testing approach

- session types: subtyping *preserves* correctness
- contracts: compliance *defines* subcontracting

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