fair termination of binary sessions

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outline

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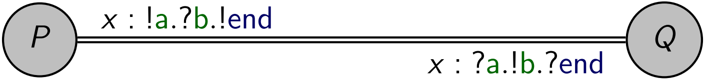
1. Introduction
2. Subtyping
3. Fair termination
4. Fair subtyping
5. Conclusion

general ideas

Definition

A **binary session** is a private communication channel linking two processes, each using one session **endpoint** according to a protocol specification called **session type**



Session types with branching points

$$?\mathsf{a}.S + ?\mathsf{b}.T \qquad\qquad !\mathsf{a}.S \oplus !\mathsf{b}.T$$

goals

- enable the **compositional static analysis** of distributed programs
- ensure that exchanged messages have the **expected type**, interactions occur in the **expected order** and processes **don't get stuck**
- **ensure that interactions terminate, eventually**

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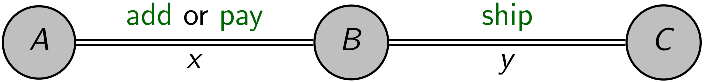
the shopper, the store and the shipper



$$
\begin{aligned}
A(x) &\triangleq \ldots\text{shopper adds items to cart and pays}\ldots \\
B(x,y) &\triangleq x?\{\mathsf{add} : B\langle x,y\rangle, \mathsf{pay} : \mathsf{wait}\ x.y!\mathsf{ship}.\mathsf{close}\ y\} \\
C(y) &\triangleq y?\mathsf{ship}.\mathsf{wait}\ y.\mathsf{done}
\end{aligned}
$$

$$(x)(A\langle x\rangle \mid (y)(B\langle x,y\rangle \mid C\langle y\rangle))$$

session type checking

- type checking ≈ matching the structure of processes and types

$$
\begin{aligned}
B(x : T, y : S) &\triangleq x?\{\mathsf{add} : B\langle x, y\rangle, \mathsf{pay} : \mathsf{wait}\ x.y!\mathsf{ship}.\mathsf{close}\ y\} \\
T &= ?\mathsf{add}.T + ?\mathsf{pay}.?\mathsf{end} \\
S &= !\mathsf{ship}.!\mathsf{end}
\end{aligned}
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\frac{}{x : T, y : S \vdash x?\{\mathsf{add} : B\langle x, y\rangle, \mathsf{pay} : \mathsf{wait}\ x.y!\mathsf{ship}.\mathsf{close}\ y\}}
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\dfrac{\dfrac{}{x : T, y : S \vdash B\langle x, y\rangle} \qquad \dfrac{\dfrac{\dfrac{}{y : !\mathsf{end} \vdash \mathsf{close}\ y}}{y : S \vdash y!\mathsf{ship}.\mathsf{close}\ y}}{x : ?\mathsf{end}, y : S \vdash \mathsf{wait}\ x.y!\mathsf{ship}.\mathsf{close}\ y}}{x : T, y : S \vdash x?\{\mathsf{add} : B\langle x, y\rangle, \mathsf{pay} : \mathsf{wait}\ x.y!\mathsf{ship}.\mathsf{close}\ y\}}
$$

parallel composition and duality

$$\dfrac{\dfrac{}{x : T, y : S \vdash B\langle x, y\rangle} \qquad \dfrac{}{y : S^{\perp} \vdash C\langle y\rangle}}{x : T \vdash (y)(B\langle x, y\rangle \mid C\langle y\rangle)}$$

- store and shipper use y according to **dual** session types

$$S = !\mathsf{ship}.!\mathsf{end} \qquad\qquad S^{\perp} = ?\mathsf{ship}.?\mathsf{end}$$

outline

one store, many shoppers

The store complies with one "canonical" protocol

$$T = ?\mathsf{add}.T + ?\mathsf{pay}.?\mathsf{end}$$

Shoppers may comply with **several** feasible protocols

$T^\perp = R = !\mathsf{add}.R \oplus !\mathsf{pay}.!\mathsf{end}$	**any number** of items
$R_1 = !\mathsf{add}.R$	**at least one** item
$R_{\mathsf{odd}} = !\mathsf{add}.(!\mathsf{add}.R_{\mathsf{odd}} \oplus !\mathsf{pay}.!\mathsf{end})$	**odd number** of items
$\ldots$	many possibilities

subtyping for session types [Gay and Hole, 2005]

$$S \leqslant T$$

Left-to-right substitution of endpoints [Liskov and Wing, 1994]

- an endpoint of type S can be safely used where an endpoint of type T is expected

$$?a \leqslant ?a + ?b \qquad !a \oplus !b \leqslant !a$$

Right-to-left substitution of processes [Gay, 2016]

- a process complying with protocol T can be safely used where a process complying with protocol S is expected

expected versus actual shopper

$$R_{\mathsf{odd}} = !\mathsf{add}.(!\mathsf{add}.R_{\mathsf{odd}} \oplus !\mathsf{pay}.!\mathsf{end}) \qquad \text{actual behavior}$$

$$T^{\perp} = R = !\mathsf{add}.R \oplus !\mathsf{pay}.!\mathsf{end} \qquad \text{expected behavior}$$

$$\dfrac{\dfrac{\overline{x : R_{\mathsf{odd}} \vdash A\langle x \rangle}}{x : T^{\perp} \vdash A\langle x \rangle}\; T^{\perp} \leqslant R_{\mathsf{odd}} \qquad \dfrac{\vdots}{x : T \vdash (y)(B\langle x, y \rangle \mid C\langle y \rangle)}}{\emptyset \vdash (x)(A\langle x \rangle \mid (y)(B\langle x, y \rangle \mid C\langle y \rangle))}$$

from safety to liveness

Theorem (nothing bad ever happens)

In a well-typed program

- *exchanged message have the expected type* *(comm. safety)*
- *interactions occur in the expected order* *(protocol fidelity)*
- *programs don't get stuck* *(deadlock freedom)*

Desideratum (something good eventually happens)

Also, in a well-typed program

- all sessions eventually terminate

outline

fair termination

Definition (fair termination)

We say that P is **fairly terminating** if each finite execution of P may be extended to done. That is

$$P \Longrightarrow Q \qquad \text{implies} \qquad Q \Longrightarrow \mathsf{done}$$

Remark

In a fairly terminating program

- every message sent may be received
- every process waiting for a message may receive one
- every session may terminate

$$P \Longrightarrow (x)(x!\mathsf{a}.P' \mid Q)$$

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why "fair termination"?

Definition (strong fairness [Francez, 1986])

A program execution is **strongly fair** if every reduction that is **infinitely often enabled** is **infinitely often performed**

Consider the shopper protocol $R = !\mathsf{add}.R \oplus !\mathsf{pay}.!\mathsf{end}$:

- an execution in which the shopper sends add forever is **unfair**
- in every maximal, fair execution the shopper eventually sends pay

Theorem

A (finite-state) program is fairly terminating if and only if every maximal, fair execution is finite and finishes with done

- In principle, a fairly-terminating program may execute forever
- In practice, *i.e.* if its realistic executions are fair, it doesn't

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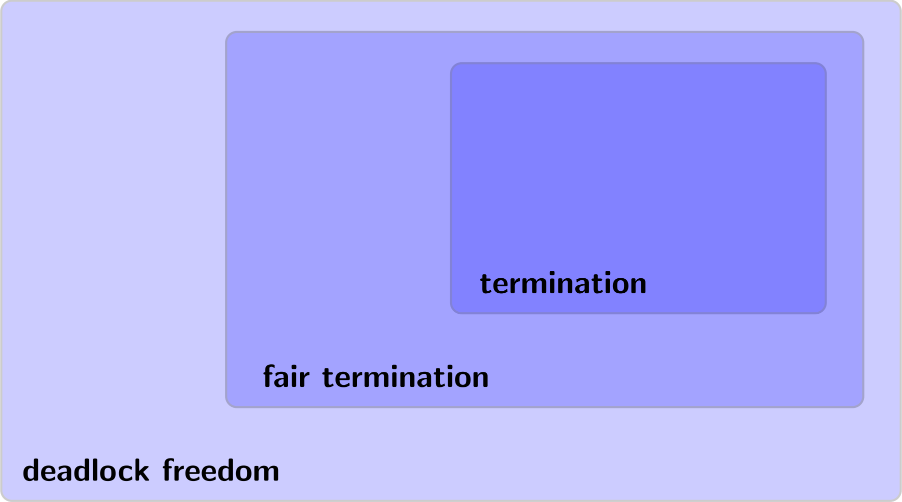
termination < fair termination < deadlock freedom



example: the compulsive shopper

$$A(x) \stackrel{\triangle}{=} x!\mathsf{add}.A\langle x\rangle \qquad R_\infty = !\mathsf{add}.R_\infty$$

$$\dfrac{\dfrac{\dfrac{}{x : R_\infty \vdash A\langle x\rangle}}{x : T^\perp \vdash A\langle x\rangle}\; T^\perp \leqslant R_\infty \qquad \dfrac{\vdots}{x : T \vdash (y)(B\langle x, y\rangle \mid C\langle y\rangle)}}{\emptyset \vdash (x)(A\langle x\rangle \mid (y)(B\langle x, y\rangle \mid C\langle y\rangle))}$$

1. this program is deadlock-free but not fairly terminating
2. the strong fairness assumption alone doesn't turn an ordinary session type system into one that ensures fair termination

example: the compulsive shopper

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$\leqslant$ is designed to preserve safety, not liveness

outline

fair subtyping [Padovani, 2013, 2016, Ciccone and Padovani, 2021a]

defined by a **generalized inference system** [Ancona et al., 2017, Dagnino, 2019]

$$\frac{}{p\,\mathsf{end} \leqslant p\,\mathsf{end}} \qquad \frac{S_k \leqslant T_k}{!\{\mathsf{a}_i : S_i\}_{i\in I} \leqslant !\{\mathsf{a}_j : T_j\}_{j\in J}} \text{ corule}$$

$$\frac{S_i \leqslant T_i \ ^{(i\in I)}}{?\{\mathsf{a}_i : S_i\}_{i\in I} \leqslant ?\{\mathsf{a}_i : T_i\}_{i\in I\cup J}} \qquad \frac{S_i \leqslant T_i \ ^{(i\in I)}}{!\{\mathsf{a}_i : S_i\}_{i\in I\cup J} \leqslant !\{\mathsf{a}_i : T_i\}_{i\in I}}$$

We say that S is a **fair subtype** of T, notation $S \leqslant T$, if

- there is an **arbitrary** derivation of $S \leqslant T$ **using just rules**
- there is a **finite** derivation $S \leqslant T$ **using rules and corules**

example of fair subtyping

$$R = !\mathsf{add}.R \oplus !\mathsf{pay}.!\mathsf{end} \qquad R_1 = !\mathsf{add}.R$$

$$\dfrac{\dfrac{\dfrac{\vdots}{R \leqslant R} \qquad \dfrac{}{!\mathsf{end} \leqslant !\mathsf{end}}}{R \leqslant R}}{R \leqslant R_1} \qquad\qquad \dfrac{\dfrac{\dfrac{}{!\mathsf{end} \leqslant !\mathsf{end}}}{R \leqslant R}}{R \leqslant R_1}$$

$$R \leqslant R_1$$

Note

- there is no finite derivation for $R \leqslant R_1$ without the corule

example of **unfair** subtyping

$R = {!}\mathsf{add}.R \oplus {!}\mathsf{pay}.{!}\mathsf{end}$ $\qquad$ $R_\infty = {!}\mathsf{add}.R_\infty$

$$\dfrac{\dfrac{\vdots}{R \leqslant R_\infty}}{R \leqslant R_\infty} \qquad\qquad \dfrac{\dfrac{\vdots}{R \leqslant R_\infty}}{R \leqslant R_\infty}$$

$$R \not\leqslant R_\infty$$

Note

- there is no finite derivation for $R \leqslant R_1$, **even with the corule**

properties of fair subtyping

Fact

Fair subtyping is a **liveness-preserving** subtyping relation closely related to *fair testing* [Natarajan and Cleaveland, 1995] and *should testing* [Rensink and Vogler, 2007]

- see Bugliesi et al. [2009], Bravetti and Zavattaro [2009], Padovani [2013, 2016], Bravetti et al. [2021] for details
- see Ciccone and Padovani [2021a] for an Agda formalization

compulsive shopping is not allowed...

$$\dfrac{\dfrac{\dfrac{}{x : R_\infty \vdash A\langle x\rangle}}{\color{red}{x : T^\perp \vdash A\langle x\rangle}}\ \cancel{T^\perp \leqslant R_\infty} \qquad \dfrac{\vdots}{x : T \vdash (y)(B\langle x, y\rangle \mid C\langle y\rangle)}}{\emptyset \vdash (x)(A\langle x\rangle \mid (y)(B\langle x, y\rangle \mid C\langle y\rangle))}$$

compulsive shopping is not allowed... or is it?

A different typing derivation for the compulsive shopper

$$A(x) \triangleq x!\mathsf{add}.A\langle x\rangle$$
$$R = !\mathsf{add}.R \oplus !\mathsf{pay}.!\mathsf{end}$$
$$R_1 = !\mathsf{add}.R$$

$$\dfrac{\dfrac{\overline{x : R \vdash A\langle x\rangle}}{x : R_1 \vdash x!\mathsf{add}.A\langle x\rangle}}{x : R \vdash x!\mathsf{add}.A\langle x\rangle}\ R \leqslant R_1$$

- $R \leqslant R_1$ is used in the typing derivation of a recursive process
- "infinitely many" usages of fair subtyping ($R \leqslant R_1$) may have the same overall effect of unfair subtyping ($R \leqslant R_\infty$)
- well-typed processes should only be allowed to perform a **bounded number of casts**

cast boundedness

$$\Gamma \vdash^n P$$

- P is well-typed in Γ and has **rank** n
- n is an upper bound to the number of casts performed by P

$$\dfrac{\dfrac{\overline{x : R \vdash^{n} A\langle x\rangle}}{x : R_1 \vdash^{n} x!\mathsf{add}.A\langle x\rangle}}{x : R \vdash^{n+1} x!\mathsf{add}.A\langle x\rangle}\ R \leqslant R_1$$

- we cannot assign a finite rank to this typing derivation

fair termination, at last

Theorem

If $\emptyset \vdash^n P$, then P is fairly terminating

Proof idea.

Show that typing is preserved by reductions (subject reduction):

- if $\Gamma \vdash^n P$ and $P \longrightarrow Q$, then $\Gamma \vdash^n Q$

Define a **measure** for well-typed processes that includes n as well as the effort required to terminate all open sessions:

- $\Gamma \vdash^{\mu} P$

Show that for every non-terminated, well-typed program **there exists** a reduct with a **strictly smaller** measure:

- if $\emptyset \vdash^{\mu} P$, either $P = \mathsf{done}$ or $P \longrightarrow Q$ and $\emptyset \vdash^{\nu} Q$ where $\nu < \mu$

Note: the measure **may increase** if new sessions are opened. □

outline

summary

A compositional static analysis ensuring fair termination

- well-typed programs terminate, if the fairness assumption is true
- infinite executions are possible, but only in principle

A nice application of generalized inference systems

- definition of fair subtyping
- typing corules (not discussed in this talk, see below)

Want more?

- many simplifications in this talk
- see Ciccone and Padovani [2021b] for details
  (higher-order sessions, proofs, type checking algorithm, ...)

ongoing and future work

Type checker implementation

- ready, soon available on my home page

Other communication models

- multiparty sessions (straightforward)
- actors, many-to-one communications (challenging)

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thank you!

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