Session Types at the Mirror

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Poll

x is an object with methods a and b

$$x : \{a; b\}$$

Behavioral operators

external choice $+$

internal choice $\oplus$

Give a behavioral type to x

(A) $x : a + b$

(B) $x : a \oplus b$

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(A) $x : a + b$	(B) $x : a \oplus b$

$x : a + b$

- the type of x tells about what x can do
- the user of x can decide which method to invoke

Let's think of subtyping

$$x : \{a; b\} \qquad \begin{array}{ccc} \{a; b\} & <: & \{a\} \\ a + b & \preceq & a \end{array} \qquad y : \{a\}$$

How do you explain this?

$$a \preceq a + b$$

$x : a + b$

- the type of x tells about what x can do
- the user of x can decide which method to invoke

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Conclusion

$$\vdash P : \{c : \sigma\}$$

1. σ is not the type of c
2. σ is the projection of P's behavior wrt c

What if we **define** session types like this?

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2. σ is the projection of P's behavior wrt c

What if we **define** session types like this?

Projecting behavior

$$S = a?(x).x?(\mathit{title} : \mathtt{Str}).x!\mathit{price}.x?(\mathit{addr} : \mathtt{Addr}).x!\mathit{date}$$

$$B_1 = (\nu c)a!c.c!\mathit{title}.c?(\mathit{price} : \mathtt{Int}).(\nu d)b!d.d!\mathit{price}/2.d!c$$

$$B_2 = b?(y).y?(\mathit{contrib} : \mathtt{Int}).y?(z).z!\mathit{address}.z?(d : \mathtt{Date})$$

a :	$?\sigma.1$	$!\sigma.1$	
b :		$!\tau.1$	$?\tau.1$
c :	$?\mathtt{Str}.!\mathtt{Int}.?\mathtt{Addr}.!\mathtt{Date}.1$	$!\mathtt{Str}.?\mathtt{Int}.1$	$!\mathtt{Addr}.?\mathtt{Date}.1$
d :		$!\mathtt{Int}.!\rho.1$	$?\mathtt{Int}.?\rho.1$

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Projecting behavior

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Session types: syntax

σ	$::=$		**session type**
		0	(failure)
	$\mid$	1	(success)
	$\mid$	$\alpha.\sigma$	(action prefix)
	$\mid$	$\sigma + \sigma$	(external choice)
	$\mid$	$\sigma \oplus \sigma$	(internal choice)
	$\mid$	$\sigma \mid \sigma$	(composition)

α	$::=$		**action**
		$?t$	(value input)
	$\mid$	$!t$	(value output)
	$\mid$	$?\sigma$	(channel input)
	$\mid$	$!\sigma$	(delegation)

Session types: operational semantics

$$1 \xrightarrow{\checkmark} 1$$

$$\frac{\sigma \xrightarrow{\checkmark} \sigma' \qquad \tau \xrightarrow{\checkmark} \tau'}{\sigma \mid \tau \xrightarrow{\checkmark} \sigma' \mid \tau'} \qquad \frac{\sigma \xrightarrow{!\mathsf{v}} \sigma' \qquad \tau \xrightarrow{?\mathsf{v}} \tau'}{\sigma \mid \tau \longrightarrow \sigma' \mid \tau'}$$

$$\frac{\sigma \xrightarrow{!\rho} \sigma' \qquad \tau \xrightarrow{?\rho'} \tau' \qquad \rho \preceq \rho'}{\sigma \mid \tau \longrightarrow \sigma' \mid \tau'}$$

$$\frac{\sigma \xrightarrow{!\rho} \sigma' \qquad \tau \xrightarrow{?\rho'} \tau' \qquad \rho \not\preceq \rho'}{\sigma \mid \tau \longrightarrow 0}$$

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subsession

$$\frac{\sigma \xrightarrow{!\rho} \sigma' \qquad \tau \xrightarrow{?\rho'} \tau' \qquad \rho \preceq \rho'}{\sigma \mid \tau \longrightarrow \sigma' \mid \tau'}$$

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wrong type

error

Characterizing complete compositions

Definition (completeness)

σ is *complete* if $\sigma \Longrightarrow \sigma'$ implies $\sigma' \stackrel{\checkmark}{\Longrightarrow}$

Examples

$$
\begin{array}{lcl}
\texttt{!Int.1} \mid \texttt{?Real.1} & \longrightarrow & \texttt{1} \mid \texttt{1} \quad \text{YES} \\
\texttt{!Int.1} \mid \texttt{1} & \not\longrightarrow & \quad \text{NO}
\end{array}
$$

Completeness generalizes duality

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Completeness generalizes duality

Characterizing well-typed processes

What's the difference?

$$!\texttt{Int}.\mathbf{1}$$
$$!\texttt{Int}.\mathbf{0}$$

Definition (viability)

σ is *viable* if $\sigma \mid \rho$ is complete for some ρ

- viable $\sim$ "different from empty type"

Characterizing well-typed processes

What's the difference?

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Defining type equality

Definition (subtyping)

$\sigma \preceq \tau$ if $\sigma \mid \rho$ complete implies $\tau \mid \rho$ complete for all ρ

$$
\begin{array}{rcl}
1 \mid \sigma & \approx & \sigma \\
0 + \sigma & \approx & \sigma \\
\alpha.\sigma + \alpha.\tau & \approx & \alpha.\sigma \oplus \alpha.\tau \\
\\
\sigma \oplus \tau & \preceq & \sigma \\
\sigma & \not\preceq & \sigma + \tau
\end{array}
$$

reduce nondeterminism

Subtyping vs refinement

Definition (subtyping)

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$P : \sigma$ can be replaced by $Q : \tau$ (left-to-right)

$$P \mid R \quad \text{ok} \qquad \Rightarrow \qquad Q \mid R \quad \text{ok}$$

$u : \tau$ can be replaced by $v : \sigma$ (right-to-left)

$$P : \{u : \tau\} \qquad \Rightarrow \qquad P\{^{v}/_{u}\} : \{v : \tau\}$$

Subtyping vs refinement

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$u : \tau$ can be replaced by $v : \sigma$ (right-to-left)

$$P : \{u : \tau\} \qquad \Rightarrow \qquad P\{{}^{v}/_{u}\} : \{v : \tau\}$$

P keeps behaving as τ

$\preceq$ is not a precongruence

$0 \approx \sigma$ if σ is not viable

- 0 is not observable
- σ may be observable
  (a faulty process may send/receive messages)

Definition (strong subtyping)

Let $\sqsubseteq$ be the largest precongruence included in $\preceq$

Theorem

$\sigma \preceq \tau$ if and only if either σ is not viable or $\sigma \sqsubseteq \tau$

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Parallel composition

t-par

$$\frac{\Gamma \vdash P : \{u_i : \sigma_i\ {}^{i \in I}\} \qquad \Gamma \vdash Q : \{u_i : \tau_i\ {}^{i \in I}\}}{\Gamma \vdash P \mid Q : \{u_i : \sigma_i \mid \tau_i\ {}^{i \in I}\}}$$

Parallel composition

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Delegation

P keeps using *v*

t-outputS

$$\frac{\Gamma \vdash P : \Delta \cup \{u : \sigma, v : \tau\}}{\Gamma \vdash u!v.P : \Delta \cup \{u : !\rho.\sigma, v : \tau \mid \rho\}}$$

$$a?(x).\cdots \mid b?(x).\cdots \mid (\nu c)a!c.b!c.\cdots$$

Channel input

P not allowed to use anything else. . .

t-inputS

$$\frac{\Gamma \vdash P : \{x : \rho\}}{\Gamma \vdash u?(x).P : \{u : ?\rho.1\}}$$

. . . not even u

Restriction

t-res

$$\frac{\Gamma \vdash P : \Delta \cup \{c : \sigma\} \qquad \sigma \text{ complete}}{\Gamma \vdash (\nu c)P : \Delta}$$

Subject reduction

Theorem

If $\Gamma \vdash P : \Delta$ and $P \longrightarrow Q$ and Δ viable, then $\Gamma \vdash Q : \Delta$

$$
\begin{array}{rclcl}
P & \stackrel{\text{def}}{=} & (\nu c)a!c & : & \{a : !1.1\} \\
Q & \stackrel{\text{def}}{=} & c?(x).x!3 & : & \{a : ?(!\mathtt{Int}.1).1\}
\end{array}
$$

$!1.1 \mid ?(!\mathtt{Int}.1).1$ not viable

$P \mid Q \longrightarrow (\nu c)(0 \mid c!3)$ OUCH!

Type safety

Theorem

If $\Gamma \vdash P : \Delta \cup \{c : \sigma\}$ *and* σ *complete and* $P \downarrow c$*, then* $P \longrightarrow$

- $P \downarrow c =$ "whoever owns c is immediately ready to use it"
- type system does not enforce *global* progress

Summary

- Projection

$\Rightarrow$ Session types as process terms

$\Rightarrow$ Semantically grounded theory of session types

  - Completeness $\sim$ duality
  - Viability $\sim$ well-typedness

Two questions answered

Q: What is a session type?
A: Projection of process behavior

- ccs-like formalism
- *re*use known techniques: (fair) testing semantics

Q: Process refinement *and* subtyping?
A: Same relation

- left-to-right replacement of processes
- right-to-left replacement of channels

Two questions answered

Q: What is a session type?
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Thank you.

Constrained delegation

$a!c.a!c.c?(x : \texttt{Int}).c?(y : \texttt{Bool})$
$a?(x).a?(y).y!\texttt{true}.x!3 \quad : \quad \cdots\{x : !\texttt{Int}.1, y : !\texttt{Bool}.1\}$

$\downarrow$

$c?(x : \texttt{Int}).c?(y : \texttt{Bool})$
$c!\texttt{true}.c!3$

Constrained delegation

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projection forgets dependency

$\downarrow$

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$\downarrow$

$c?(x : \texttt{Int}).c?(y : \texttt{Bool})$

$c!\texttt{true}.c!3$

OUCH!

Replication

t-bang

$$\frac{\Gamma \vdash P : \{u_i : \sigma_i\ ^{i \in I}\} \qquad \sigma_i \sqsubseteq \sigma_i \mid \sigma_i\ ^{i \in I}}{\Gamma \vdash \star P : \{u_i : \sigma_i\ ^{i \in I}\}}$$

$\sigma \sqsubseteq \sigma \mid \sigma$

- doesn't matter whether there are 1 or 2^{100} copies of P
- $\sigma = 1$ (does nothing)
- $\sigma = 1 \oplus \pi.\sigma$ (can do π at any time)

Subsumption

$$\frac{\Gamma \vdash P : \Delta \cup \{u : \tau\} \qquad \sigma \sqsubseteq \tau}{\Gamma \vdash P : \Delta \cup \{u : \sigma\}}\ \text{t-sub}$$

Terminated process

t-weak

$$\frac{\Gamma \vdash P : \Delta \qquad u \notin \mathtt{dom}(\Delta)}{\Gamma \vdash P : \Delta \cup \{u : 1\}}$$

t-nil

$$\frac{}{\Gamma \vdash 0 : \emptyset}$$

Communication

t-input

$$\frac{\Gamma, x : t \vdash P : \Delta \cup \{u : \sigma\}}{\Gamma \vdash u?(x : t).P : \Delta \cup \{u : ?t.\sigma\}}$$

t-output

$$\frac{\Gamma \vdash e : t \qquad \Gamma \vdash P : \Delta \cup \{u : \sigma\}}{\Gamma \vdash u!e.P : \Delta \cup \{u : !t.\sigma\}}$$

Choices

t-ext

$$\frac{\Gamma \vdash \pi_i.P_i : \Delta \cup \{u : \sigma_i\}^{\ i \in I}}{\Gamma \vdash \sum_{i \in I} \pi_i.P_i : \Delta \cup \{u : \sum_{i \in I} \sigma_i\}}$$

t-int

$$\frac{\Gamma \vdash P : \Delta \qquad \Gamma \vdash Q : \Delta}{\Gamma \vdash P \oplus Q : \Delta}$$