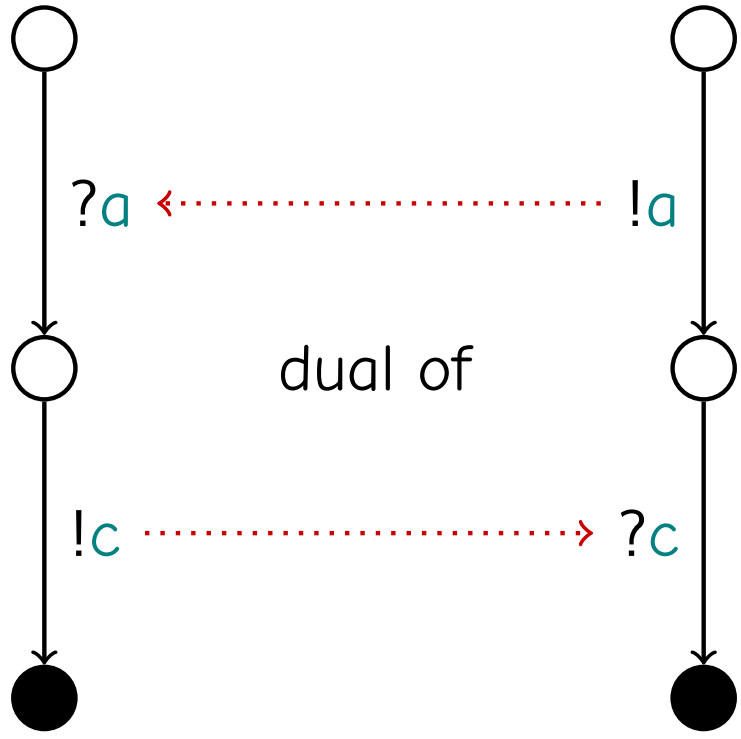
fair termination of asynchronous binary sessions

Luca Padovani Gianluigi Zavattaro

Department of Computer Science and Engineering – University of Bologna

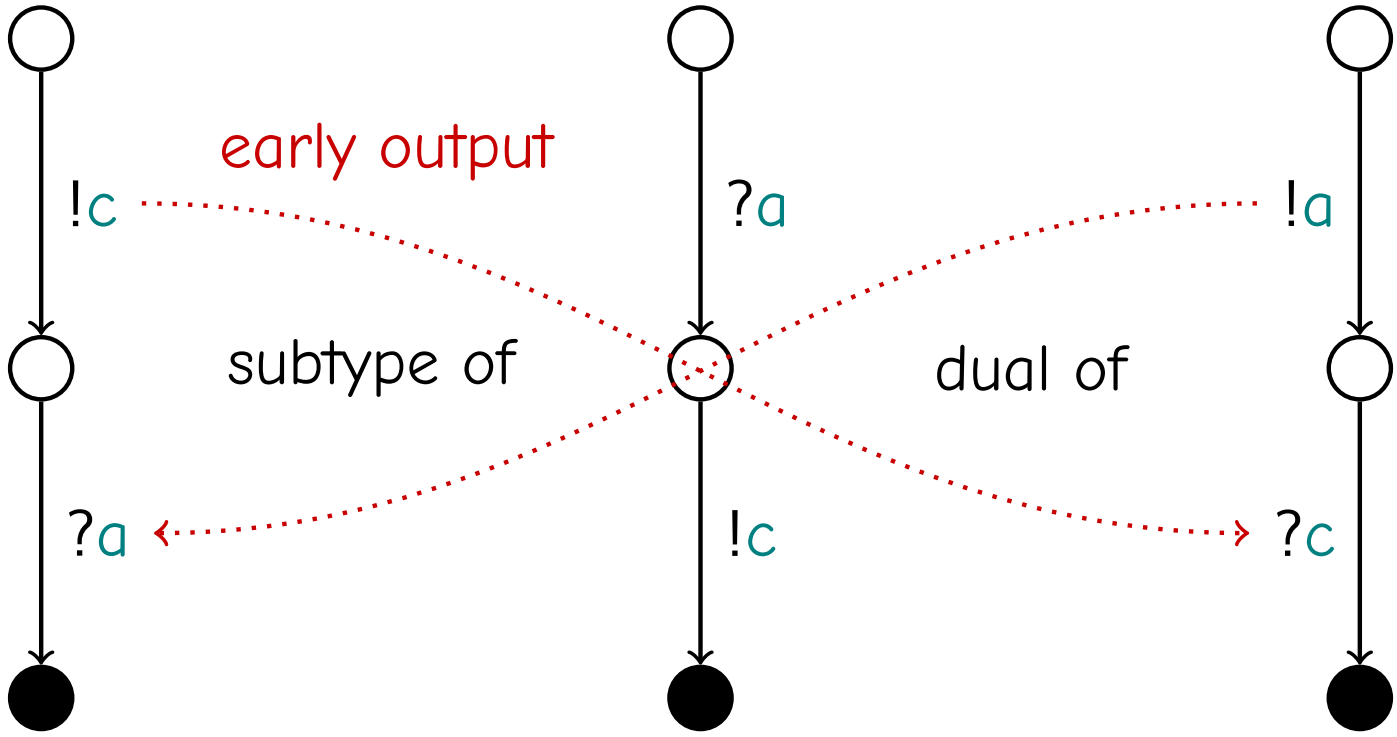
duality, asynchrony and subtyping



- communication safety
- progress
- half-duplex communication

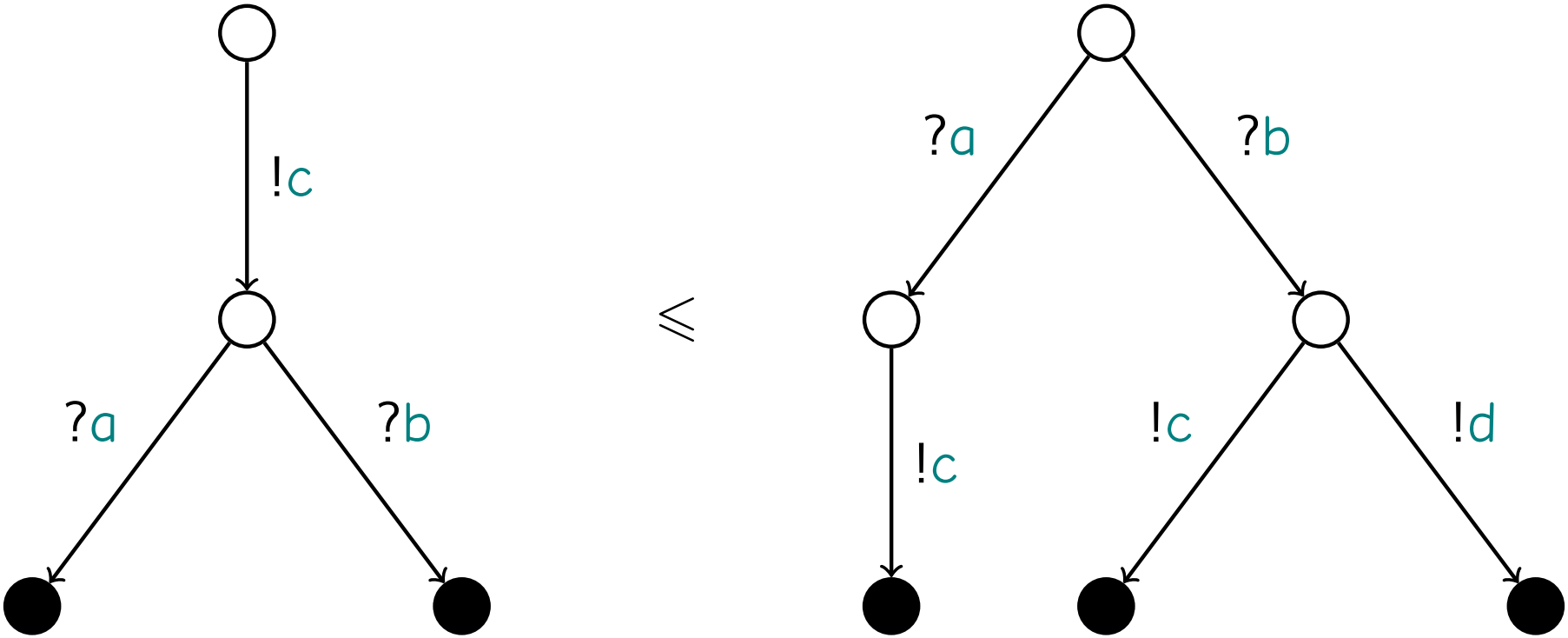
duality, asynchrony and subtyping

Mostrous et al. [2009], Chen et al. [2017], Ghilezan et al. [2023]



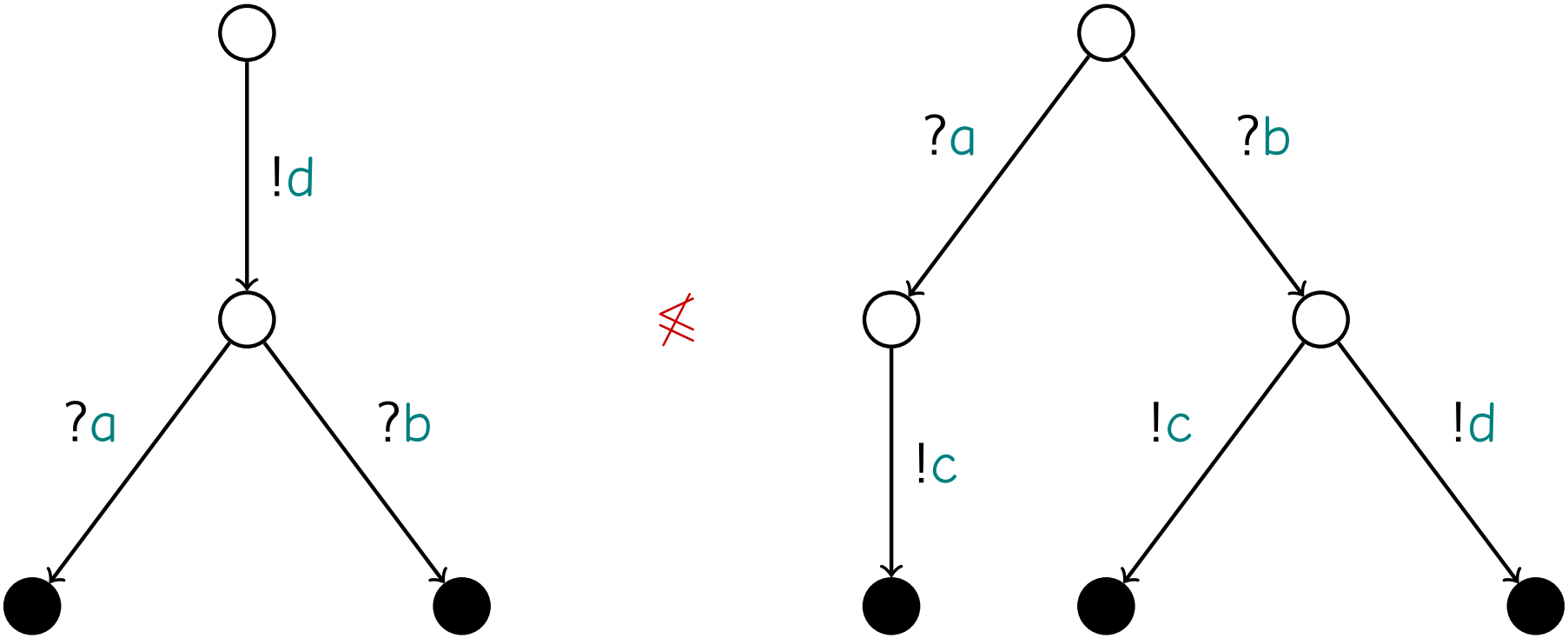
- communication safety
- progress
- full-duplex communication

output anticipation and causal dependencies



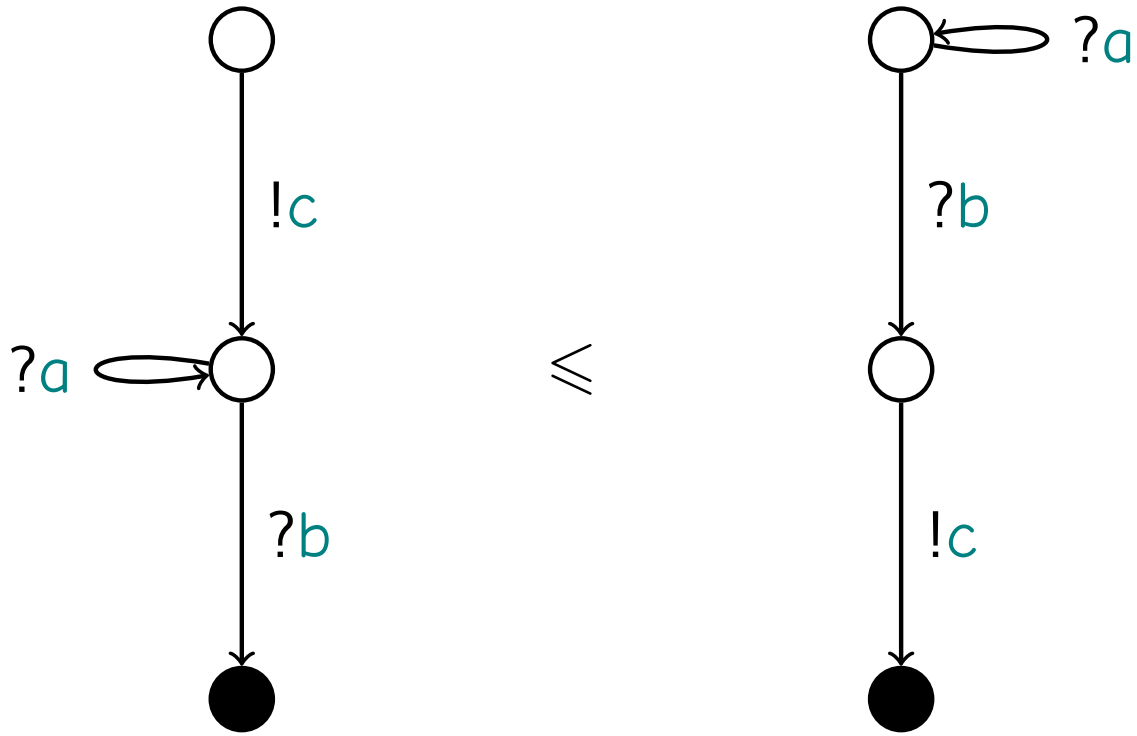
- c can be sent early (doesn't depend on the previous input)

output anticipation and causal dependencies



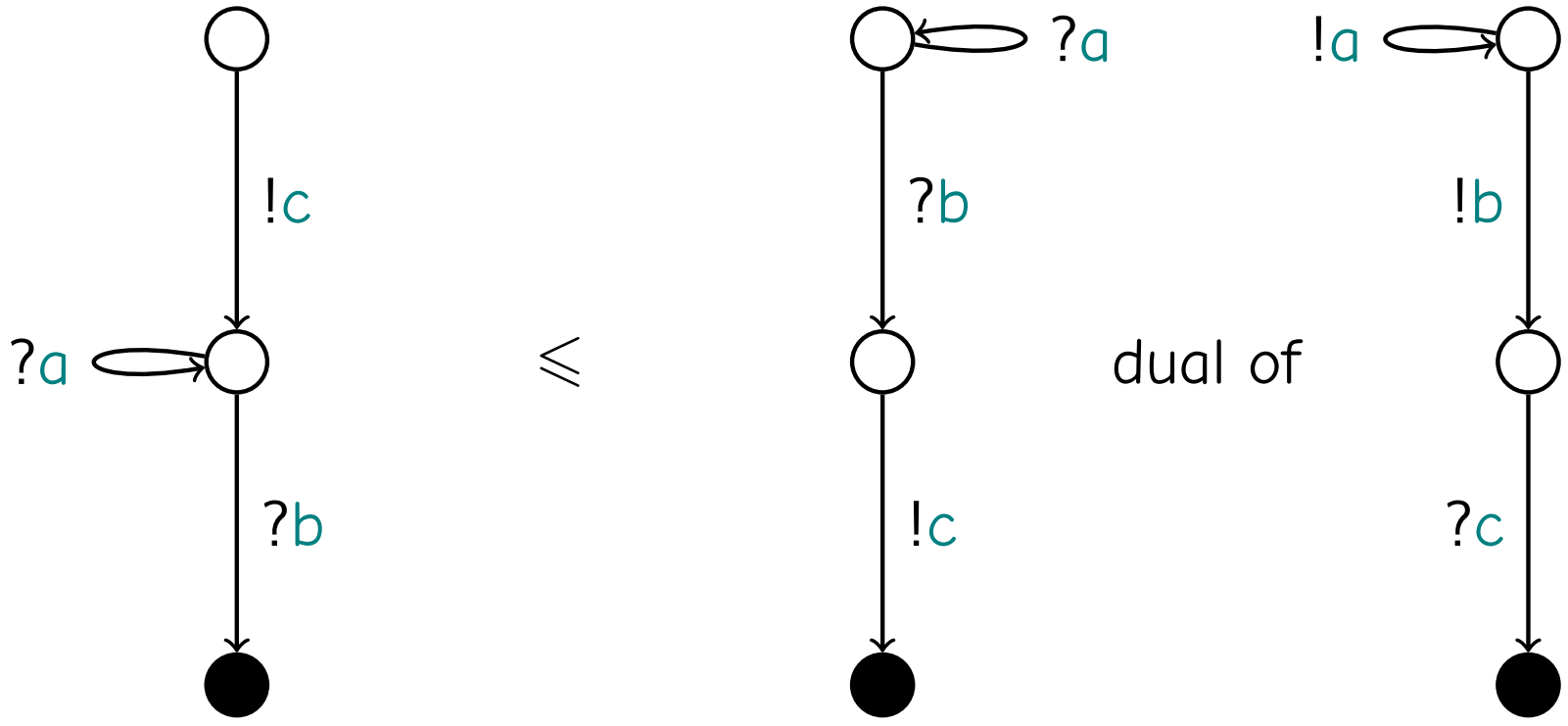
- c can be sent early (doesn't depend on the previous input)
- d cannot be sent early (depends on the previous input)

unbounded output anticipation



- aforementioned approaches allow **bounded** output anticipation
- here we consider **unbounded** output anticipation as well

unbounded output anticipation



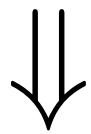
- aforementioned approaches allow **bounded** output anticipation
- here we consider **unbounded** output anticipation as well
- without a fairness assumption we can have orphan messages

contribution

fair asynchronous subtyping

+

type system ensuring weak process termination



unbounded output anticipation

no orphan messages

towards asynchronous subtyping

Our approach

- capture asynchrony in the **transition relation** of session types
- transitions say what a process can do, not what it does

$$\frac{k \in I}{\oplus\{a_i : S_i\}_{i \in I} \xrightarrow{!a_k} S_k} \qquad \frac{\forall i \in I : S_i \xrightarrow{!c} T_i}{\&\{a_i : S_i\}_{i \in I} \xrightarrow{!c} \&\{a_i : T_i\}_{i \in I}}$$

Example: deriving an **early output**

$$\frac{\dfrac{}{!c.S_1 \xrightarrow{!c} S_1} \qquad \dfrac{}{!c.S_2 \oplus !d.T \xrightarrow{!c} S_2}}{?a.!c.S_1 \,\&\, ?b.(!c.S_2 \oplus !d.T) \xrightarrow{!c} ?a.S_1 \,\&\, ?b.S_2}$$

a problem with recursive session types

$$S = ?\mathsf{a}.S \,\&\, ?\mathsf{b}.!\mathsf{c}.U \qquad T = ?\mathsf{a}.T \,\&\, ?\mathsf{b}.U$$

$$\dfrac{\dfrac{}{!\mathsf{c}.U \xrightarrow{!\mathsf{c}} U}}{S \xrightarrow{!\mathsf{c}} T}$$

a problem with recursive session types

$$S = ?\mathsf{a}.S \,\&\, ?\mathsf{b}.!\mathsf{c}.U \qquad T = ?\mathsf{a}.T \,\&\, ?\mathsf{b}.U$$

$$\frac{S \xrightarrow{!\mathsf{c}} T \qquad \overline{!\mathsf{c}.U \xrightarrow{!\mathsf{c}} U}}{S \xrightarrow{!\mathsf{c}} T}$$

a problem with recursive session types

$$S = ?\mathsf{a}.S \,\&\, ?\mathsf{b}.!\mathsf{c}.U \qquad T = ?\mathsf{a}.T \,\&\, ?\mathsf{b}.U$$

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$$\dfrac{\dfrac{\dfrac{\vdots}{S \xrightarrow{!\mathsf{c}} T} \qquad \dfrac{}{!\mathsf{c}.U \xrightarrow{!\mathsf{c}} U}}{S \xrightarrow{!\mathsf{c}} T} \qquad \dfrac{}{!\mathsf{c}.U \xrightarrow{!\mathsf{c}} U}}{S \xrightarrow{!\mathsf{c}} T}$$

a problem with recursive session types

$$S = ?a.S \,\&\, ?b.!c.U \qquad T = ?a.T \,\&\, ?b.U$$

$$\dfrac{\dfrac{\vdots}{S \xrightarrow{!c} T} \qquad \dfrac{}{!c.U \xrightarrow{!c} U}}{S \xrightarrow{!c} T} \qquad \dfrac{}{!c.U \xrightarrow{!c} U}$$

$$\dfrac{S \xrightarrow{!c} T \qquad !c.U \xrightarrow{!c} U}{S \xrightarrow{!c} T}$$

we turn the problem into the solution

- define the LTS **coinductively** 😜
- there's a catch: make sure no phony transitions are derivable
- use a **generalized inference system** [Ancona et al., 2017]

fair asynchronous subtyping

Definition (simplified, first order)

Fair asynchronous subtyping is the largest $\leqslant$ s.t. $S \leqslant T$ implies

1. $S \xrightarrow{!a} S'$ implies $T \xrightarrow{!a} T'$ and $S' \leqslant T'$
2. $T \xrightarrow{?a} T'$ implies $S \xrightarrow{?a} S'$ and $S' \leqslant T'$

Properties

- *"same" as synchronous subtyping* *(input/output variance)*

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- *"same" as synchronous subtyping* *(input/output variance)*
- $S \leqslant T$ *implies* $T^{\perp} \leqslant S^{\perp}$ *(closure under duality)*

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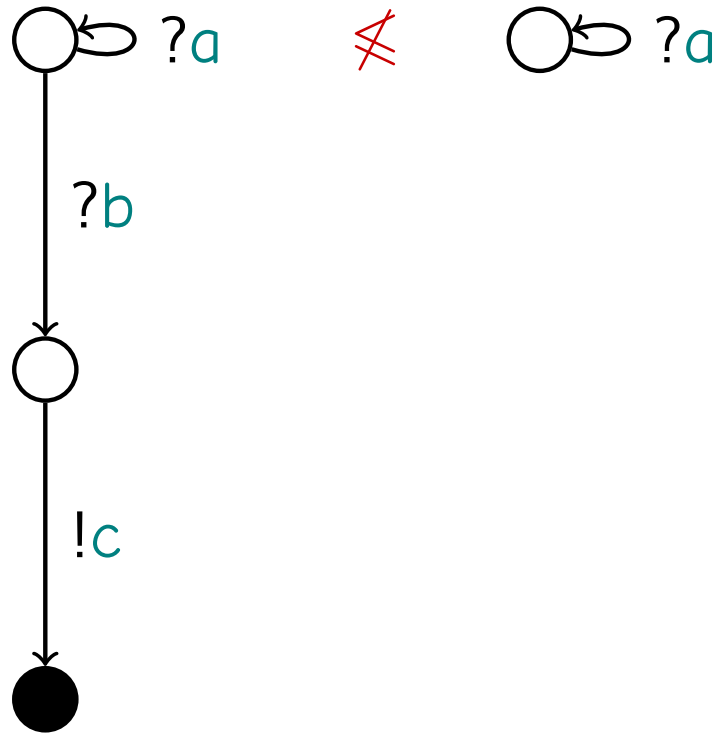
Properties

- *"same" as synchronous subtyping* *(input/output variance)*
- $S \leqslant T$ *implies* $T^{\perp} \leqslant S^{\perp}$ *(closure under duality)*
- *undecidable*

some corner cases



- holds for synchronous subtyping [Gay and Hole, 2005]

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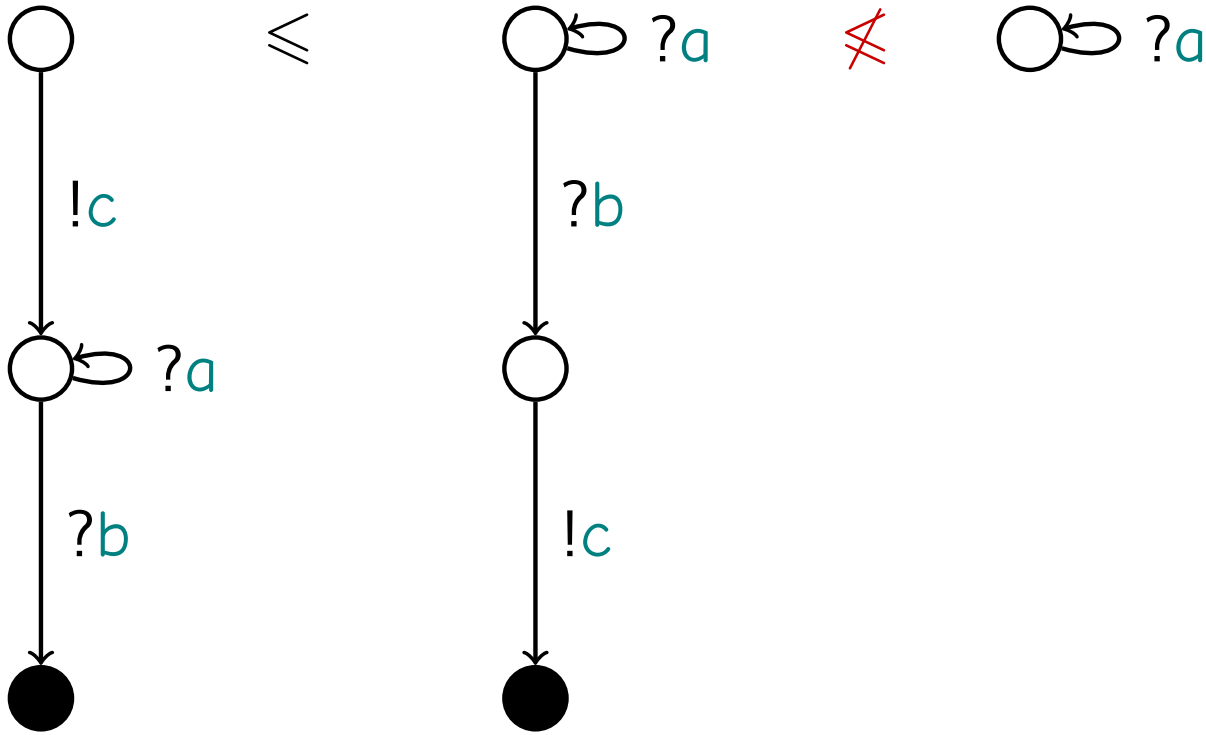

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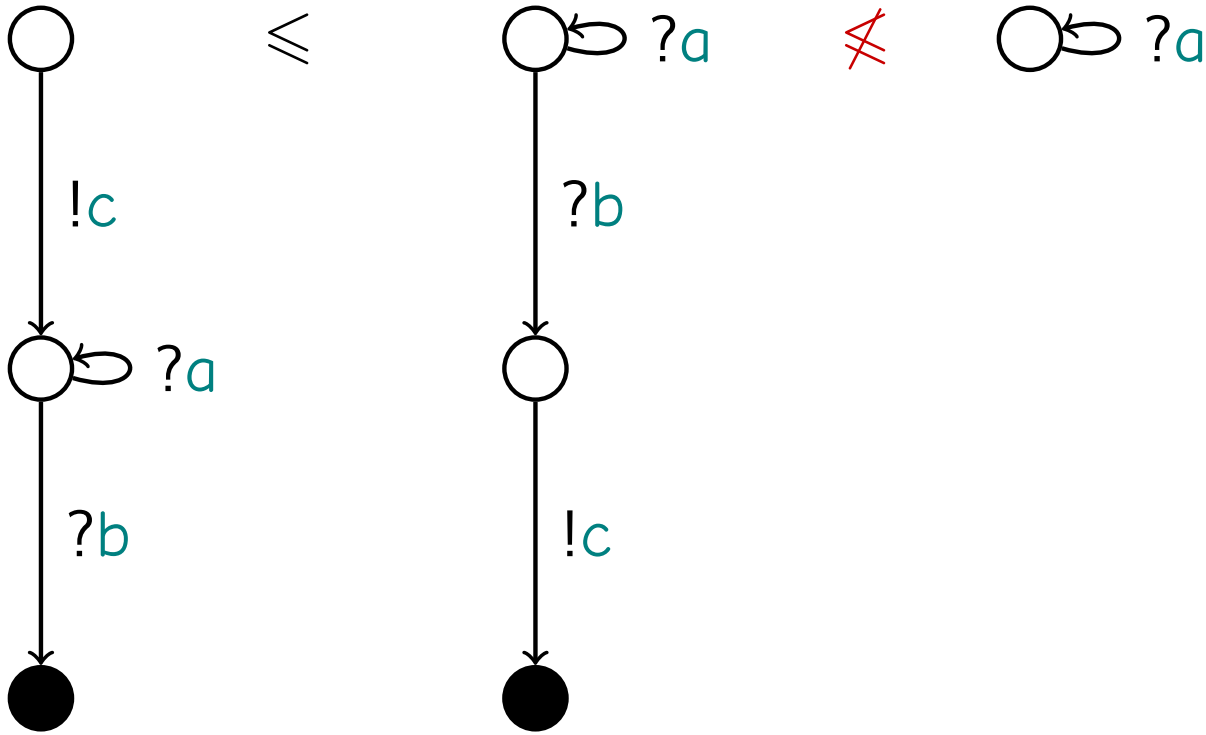

- holds for synchronous subtyping [Gay and Hole, 2005]
- no big deal, ↺ ?a is not inhabited in our type system

ensuring weak program termination

Starting point: type system based on **linear logic**

- (in)finitary logic with fixed points *e.g.* [Doumane, 2017]
- infinitary logic with measures [Dagnino and Padovani, 2024]

heavier on annotations but simpler proofs

(weak) cut elimination $\Rightarrow$ (weak) program termination

...but what about asynchrony?

asynchrony in a logical setting

Queueless asynchronous semantics: add "deep" cut reductions

$$(x)(x \lhd c.P \mid \quad x \rhd c.Q\ ) \to (x)(P \mid \quad Q\ )$$

asynchrony in a logical setting

Queueless asynchronous semantics: add "deep" cut reductions

$$(x)(x \lhd c.P \mid \mathcal{B}_x[x \rhd c.Q]) \to (x)(P \mid \mathcal{B}_x[Q])$$

sequence of output prefixes on x

asynchrony in a logical setting

Queueless asynchronous semantics: add "deep" cut reductions

$$(x)(x \lhd c.P \mid \mathcal{B}_x[x \rhd c.Q]) \to (x)(P \mid \mathcal{B}_x[Q])$$

sequence of output prefixes on x

Asynchronous subtyping: use **explicit coercions**

classical linear logic

$$\frac{S^\perp = T}{x \leftrightarrow y \vdash x : S, y : T}$$

our type system

$$\frac{S^\perp \leqslant T}{x \leftrightarrow y \vdash x : S, y : T}$$

(closure under duality)

properties of well-typed processes

The usual stuff

- subject reduction
- deadlock freedom
- weak termination

In addition

- if $P \vdash \emptyset$, then P is **orphan message free**

Proof

- Suppose $P \to^* Q$ where Q contains some floating message c
- $Q \vdash \emptyset$ (subject reduction)
- $Q \to^*$ done (no deadlocks & weak termination)
- c must have been consumed (messages don't vanish)

concluding remarks

Fair asynchronous subtyping

- coinductive LTS
- simple characterization *à la* Gay and Hole [2005]
- nice properties (variance, closure, unbounded anticipation)

Type system

- based on "asynchronous" linear logic
- ensures weak termination hence absence of orphan messages

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Conjecture

- easy extension to multiparty sessions

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thank you

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