

Deadlock and lock freedom in the linear π -calculus

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Outline

- ① Introduction
- ② The linear π -calculus
- ③ Examples
- ④ Concluding remarks

Progress in binary systems

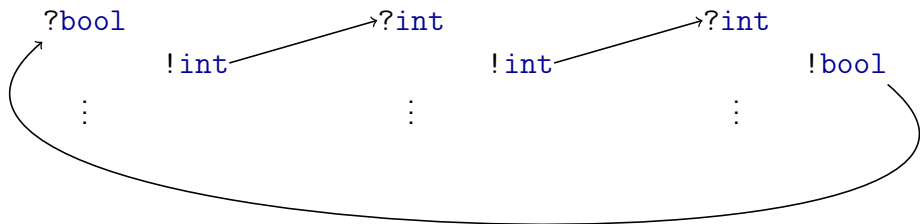
$a : T$		$a : \overline{T}$
$!int$	\rightarrow	$?int$
$!int$	\rightarrow	$?int$
$?bool$	\leftarrow	$!bool$
\vdots	\vdots	\vdots

From binary to n -ary systems

$a : T_1 \quad b : \bar{T}_2$

$b : T_2 \quad c : \bar{T}_3$

$c : T_3 \quad a : \bar{T}_1$



From binary to multiparty session types

- Bravetti, Zavattaro, **A Foundational Theory of Contracts for Multi-party Service Composition**, Fundamenta Informaticae 2008
- Honda, Yoshida, Carbone, **Multiparty asynchronous session types**, POPL 2008
- ...

$$G = A \xrightarrow{\text{int}} B.B \xrightarrow{\text{int}} C.C \xrightarrow{\text{bool}} A.G$$

~~$$A \xrightarrow{\text{int}} B + C \xrightarrow{\text{bool}} D$$~~

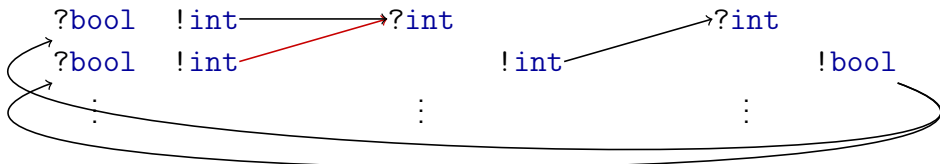
Well-formed global type \Rightarrow progress

$$G = A \xrightarrow{\text{int}} B.B \xrightarrow{\text{int}} C.C \xrightarrow{\text{bool}} A.G$$

$a : T_1 \quad b : \bar{T}_2$

$b : T_2 \quad c : \bar{T}_3$

$c : T_3 \quad a : \bar{T}_1$

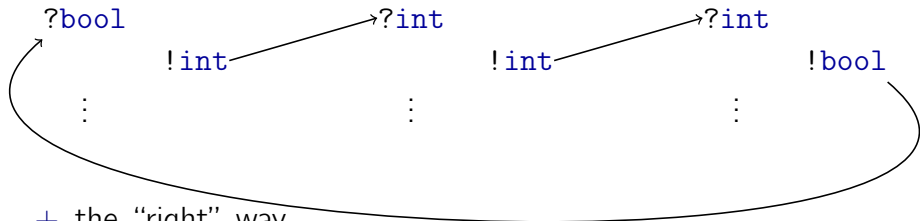


Tracking dependencies between sessions

- Dezani, de'Liguoro, Yoshida, **On Progress for Structured Communications**, TGC 2007
- Bettini, Coppo, D'Antoni, De Luca, Dezani, Yoshida, **Global Progress in Dynamically Interleaved Multiparty Sessions**, CONCUR 2008
- Coppo, Dezani, Padovani, Yoshida, **Inference of Global Progress Properties for Dynamically Interleaved Multiparty Sessions**, COORDINATION 2013
- Coppo, Dezani, Yoshida, Padovani, **Global Progress in Dynamically Interleaved Multiparty Sessions**, MSCS, to appear

Tracking dependencies between **sessions**

$a : T_1 \quad b : \bar{T}_2 \qquad b : T_2 \quad c : \bar{T}_3 \qquad c : T_3 \quad a : \bar{T}_1$



+ the “right” way

– coarse-grained

– recursion+session interleaving

Tracking dependencies between sessions

$$\begin{array}{c}
 \frac{\Gamma, u : S \vdash u : S \text{ (NAME)} \quad \Gamma \vdash \text{true, false} : \text{bool}}{\Gamma \vdash \mathbb{T}[p](y). P \triangleright \Delta} \text{ (MCAST)} \quad \frac{\Gamma \vdash e_1 : \text{bool} \quad (i = 1, 2)}{\Gamma \vdash e_1 \text{ and } e_2 : \text{bool}} \text{ (BOOL), (AND)} \\
 \frac{\Gamma \vdash e : S \quad \Gamma \vdash P \triangleright \Delta, c : T}{\Gamma \vdash c!(\Pi, e). P \triangleright \Delta, c : !(\Pi, T). T, c' : T} \text{ (SEND)} \quad \frac{\Gamma, x : S \triangleright P \triangleright \Delta, c : T}{\Gamma \vdash c?(q, x). P \triangleright \Delta, c : ?(q, S). T} \text{ (RCV)} \\
 \frac{\Gamma \vdash P \triangleright \Delta, c : T}{\Gamma \vdash c!(\langle p, e' \rangle). P \triangleright \Delta, c : !(\langle p, T \rangle). T, c' : T} \text{ (DDELEG)} \quad \frac{\Gamma \vdash P \triangleright \Delta, c : T, y : T}{\Gamma \vdash c?(q, y). P \triangleright \Delta, c : ?(q, T). T} \text{ (SRCV)} \\
 \frac{\Gamma \vdash P \triangleright \Delta, c : T_j \quad j \in I}{\Gamma \vdash c @ (\Pi, I_j). P \triangleright \Delta, c : @ (\Pi, \{I_j\}_{j \in I})} \text{ (SIL)} \quad \frac{\Gamma \vdash P \triangleright \Delta, c : T_j \quad j \in I}{\Gamma \vdash c \& (p, \{I_j\}_{j \in I}). P \triangleright \Delta, c : \& (p, \{I_j\}_{j \in I})} \text{ (BRANCH)} \\
 \frac{\Gamma \vdash P \triangleright \Delta \quad \Gamma \vdash Q \triangleright \Delta'}{\Gamma \vdash P \mid Q \triangleright \Delta, \Delta'} \text{ (PAR)} \quad \frac{\Gamma \vdash e : \text{bool} \quad \Gamma \vdash P \triangleright \Delta \quad \Gamma \vdash Q \triangleright \Delta}{\Gamma \vdash \text{if } e \text{ then } P \text{ else } Q \triangleright \Delta} \text{ (IF)} \\
 \frac{\Delta \text{ end only}}{\Gamma \vdash \mathbf{0} \triangleright \Delta} \text{ (INACT)} \quad \frac{\Gamma, a : G \triangleright P \triangleright \Delta}{\Gamma \vdash (va : G) P \triangleright \Delta} \text{ (NRES)} \\
 \frac{\Gamma \vdash e : S \quad \Delta \text{ end only}}{\Gamma, X : S \text{ T} \vdash X(e, c) \triangleright \Delta, c : T} \text{ (VAR)} \quad \frac{\Gamma, X : S \text{ T}, x : S \triangleright P \triangleright y : T \quad \Gamma, X : S \text{ T} \text{ L} \text{ T} \triangleright Q \triangleright \Delta}{\Gamma \vdash \text{def } X(x, y) = P \text{ in } Q \triangleright \Delta} \text{ (DEF)}
 \end{array}$$

$$\begin{array}{c}
 \frac{\Theta, \mathcal{A}, \mathcal{A}' ; \mathcal{S} \vdash P \triangleright \mathcal{S} \quad a \in \mathcal{A}}{\Theta, \mathcal{A}, \mathcal{A}' ; \mathcal{S} \vdash a[p](y). P \triangleright \mathcal{S} \setminus \{a\} y} \text{ (INTR)} \quad \frac{\Theta, \mathcal{A}, \mathcal{A}' ; \mathcal{S} \vdash P \triangleright \mathcal{S} \quad a \in \mathcal{A}'}{\Theta, \mathcal{A}, \mathcal{A}' ; \mathcal{S} \vdash a[p](y). P \triangleright \mathcal{S} \setminus y} \text{ (INTRN)} \\
 \frac{\Theta, \mathcal{A}, \mathcal{A}' ; \mathcal{S} \vdash P \triangleright \mathcal{S} \quad a \in \mathcal{S} \quad \text{fc}(P) \subseteq y}{\Theta, \mathcal{A}, \mathcal{A}' ; \mathcal{S} \vdash a[p](y). P \triangleright \mathcal{S} \setminus y} \text{ (INTRB)} \quad \frac{\Theta, \mathcal{A}, \mathcal{A}' ; \mathcal{S} \vdash P \triangleright \mathcal{S} \quad \text{fc}(P) \subseteq y}{\Theta, \mathcal{A}, \mathcal{A}' ; \mathcal{S} \vdash x[p](y). P \triangleright \mathcal{S} \setminus y} \text{ (INTRV)} \\
 \frac{\Theta, \mathcal{A}, \mathcal{A}' ; \mathcal{S} \vdash P \triangleright \mathcal{S} \quad e \in \mathcal{S} \Rightarrow e \in \mathcal{A} \cup \mathcal{S}}{\Theta, \mathcal{A}, \mathcal{A}' ; \mathcal{S} \vdash c!(\Pi, e). P \triangleright \mathcal{S}} \text{ (SEND)} \quad \frac{\Theta, \mathcal{A}, \mathcal{A}' ; \mathcal{S} \vdash P \triangleright \mathcal{S}}{\Theta, \mathcal{A}, \mathcal{A}' ; \mathcal{S} \vdash P \triangleright \mathcal{S}} \text{ (RCV)} \\
 \frac{\Theta, \mathcal{A}, \mathcal{A}' ; \mathcal{S} \vdash P \triangleright \mathcal{S}}{\Theta, \mathcal{A}, \mathcal{A}' ; \mathcal{S} \vdash c!(\langle p, e' \rangle). P \triangleright \{(\lambda(c) \prec \lambda(c')) \cup \mathcal{S}\}^+} \text{ (DDELEG)} \quad \frac{\Theta, \mathcal{A}, \mathcal{A}' ; \mathcal{S} \vdash P \triangleright \mathcal{S} \quad \mathcal{S} \setminus \mathcal{S} \subseteq \{\lambda(c) \prec y\}}{\Theta, \mathcal{A}, \mathcal{A}' ; \mathcal{S} \vdash c?(q, x). P \triangleright \{ \text{pre}(c, \text{fc}(P)) \cup \mathcal{S} \}^+} \text{ (SRCV)} \\
 \frac{}{\Theta, \mathcal{A}, \mathcal{A}' ; \mathcal{S} \vdash \mathbf{0} \triangleright \mathbf{0}} \text{ (INACT)} \quad \frac{\Theta, \mathcal{A}, \mathcal{A}' ; \mathcal{S} \vdash P \triangleright \mathcal{S} \quad a \in \mathcal{S}}{\Theta, \mathcal{A}, \mathcal{A}' ; \mathcal{S} \setminus \{a\} \vdash (va : G) P \triangleright \mathcal{S} \setminus \{a\}} \text{ (NRES)} \\
 \frac{\Theta, \mathcal{A}, \mathcal{A}' ; \mathcal{S} \vdash P_1 \triangleright \mathcal{S}_1 \quad \Theta, \mathcal{A}, \mathcal{A}' ; \mathcal{S} \vdash P_2 \triangleright \mathcal{S}_2}{\Theta, \mathcal{A}, \mathcal{A}' ; \mathcal{S} \vdash P_1 \mid P_2 \triangleright \{\mathcal{S}_1 \cup \mathcal{S}_2\}^+} \text{ (PAR)} \quad \frac{\Theta, \mathcal{A}, \mathcal{A}' ; \mathcal{S} \vdash P_1 \triangleright \mathcal{S}_1 \quad \Theta, \mathcal{A}, \mathcal{A}' ; \mathcal{S} \vdash P_2 \triangleright \mathcal{S}_2}{\Theta, \mathcal{A}, \mathcal{A}' ; \mathcal{S} \vdash \text{if } e \text{ then } P_1 \text{ else } P_2 \triangleright \{\mathcal{S}_1 \cup \mathcal{S}_2\}^+} \text{ (IF)} \\
 \frac{\Theta, \mathcal{A}, \mathcal{A}' ; \mathcal{S} \vdash P \triangleright \mathcal{S}}{\Theta, \mathcal{A}, \mathcal{A}' ; \mathcal{S} \vdash c @ (\Pi, I_j). P \triangleright \mathcal{S}} \text{ (SIL)} \quad \frac{\Theta, \mathcal{A}, \mathcal{A}' ; \mathcal{S} \vdash P \triangleright \mathcal{S} \quad \forall i \in I}{\Theta, \mathcal{A}, \mathcal{A}' ; \mathcal{S} \vdash c \& (p, \{I_j\}_{j \in I}) \triangleright \{ \text{pre}(c, \bigcup_{i \in I} \text{fc}(P_i)) \cup \bigcup_{i \in I} \mathcal{S}_i \}^+} \text{ (BRANCH)} \\
 \frac{e \in \mathcal{S} \Rightarrow e \in \mathcal{A} \cup \mathcal{S}}{\Theta, X[y] \triangleright \mathcal{S}; \mathcal{A}, \mathcal{A}' ; \mathcal{S} \vdash P \triangleright \mathcal{S} \quad \Theta, X[y] \triangleright \mathcal{S}; \mathcal{A}, \mathcal{A}' ; \mathcal{S} \vdash X(e, c) \triangleright \mathcal{S} \setminus \{\lambda(c), y\}} \text{ (VAR)} \\
 \frac{\Theta, X[y] \triangleright \mathcal{S}; \mathcal{A}, \mathcal{A}' ; \mathcal{S} \vdash P \triangleright \mathcal{S} \quad \Theta, X[y] \triangleright \mathcal{S}; \mathcal{A}, \mathcal{A}' ; \mathcal{S} \vdash Q \triangleright \mathcal{S}}{\Theta, \mathcal{A}, \mathcal{A}' ; \mathcal{S} \vdash \text{def } X(x, y) = P \text{ in } Q \triangleright \mathcal{S}} \text{ (DEF)}
 \end{array}$$

Tracking dependencies between **actions**

- Kobayashi, **A Type System for Lock-Free Processes**, Inf. & Comp. 2002
 - Kobayashi, **A New Type System for Deadlock-Free Processes**, CONCUR 2006
 - ...
 - Padovani, **From Lock Freedom to Progress Using Session Types**, PLACES 2013
 - Vieira, Vasconcelos, **Typing Progress in Communication-Centred Systems**, COORDINATION 2013
- + fine-grained
- recursion+channel interleaving

From sessions to the linear π -calculus

- Dardha, Giachino, Sangiorgi, **Session types revisited**, PPDP 2012

Session

$a!\langle 45 \rangle . a?(x)$

$a : !\text{int} . ?\text{bool}$

Linear π -calculus

$(\nu b) a!\langle 45, b \rangle . b?(x)$

$a : ![\text{int} \times ![\text{bool}]]$

The linear π -calculus

- Kobayashi, Pierce, Turner, **Linearity and the pi-calculus**, TOPLAS 1999

$$p^t[t]$$

Theorem (soundness)

Each *linear channel* of a well-typed process is used *at most* once

Deadlock and lock freedom

Definition (deadlock freedom)

no pending communications on linear channels in irreducible states

$$a?(x).b!\langle x \rangle \mid b?(y).a!\langle y \rangle$$

Definition (lock freedom)

each pending communication on a linear channel can be completed

$$c!\langle a \rangle \mid *c?(x).c!\langle x \rangle \mid a!\langle 1984 \rangle$$

$$\begin{array}{l}
 t ::= \text{int} \\
 | \alpha \\
 | \mu\alpha.t \\
 | t \times s \\
 | p^l[t]^{m,n}
 \end{array}$$

+ priority (\Leftarrow deadlock freedom)

Conditions for deadlock freedom

Inputs

$$\frac{u \text{ has higher priority than all the channels in } P}{u?(x).P \text{ is well typed}}$$

Outputs

$$\frac{u \text{ has higher priority than } v}{u!\langle v \rangle \text{ is well typed}}$$

Examples

$$a : \#[\text{int}]^m, b : \#[\text{bool}]^h \vdash a^m?(x).b^h!\langle x \rangle \mid b^h?(y).a^m!\langle y \rangle$$

$$a : \#[\text{int}]^m \vdash a^m?(x).a^m!\langle x \rangle$$

$$a : \#[\mu\alpha.?[\alpha]^m]^m \vdash a^m!\langle a^m \rangle$$

Recursive processes

$*fact?(x, y^0). \text{if } x = 0 \text{ then } y^0! \langle 1 \rangle$
 $\text{else } (\nu a^{-1})(fact! \langle x - 1, a^{-1} \rangle \mid a^{-1}?(z). y^0! \langle x \times z \rangle)$

- Kobayashi, **A New Type System for Deadlock-Free Processes**, CONCUR 2006

$*stream?(x, y^0). (\nu a^1)(y^0! \langle x, a^1 \rangle \mid stream! \langle x + 1, a^1 \rangle)$

Spot the differences

$c?(x^8).y^7!\langle x^8 \rangle$

- y free
- x 's priority $\prec 7$
- c monomorphic

$c?(x^8, y^7).y^7!\langle x^8 \rangle$

- y bound
- x 's priority $\prec y$'s priority
- c **polymorphic**

When is a channel polymorphic?

- input on c has free linear channels in continuation
 $\Rightarrow c$ monomorphic
- input on c has no free linear channels in continuation
 $\Rightarrow c$ polymorphic

$$\frac{\Gamma, x : t \vdash P \quad \text{un}(\Gamma)}{\Gamma \vdash *u?(x).P}$$

Fact

Every replicated channel is polymorphic in the linear π -calculus

Back to *fact* and *stream*

$*fact?(x, y^0). \text{if } x = 0 \text{ then } y^0! \langle 1 \rangle$
 $\quad \text{else } (\nu a^{-1})(fact! \langle x - 1, a^{-1} \rangle \mid a^{-1}?(z).y^0! \langle x \times z \rangle)$

$*stream?(x, y^0). (\nu a^1)(y^0! \langle x, a^1 \rangle \mid stream! \langle x + 1, a^1 \rangle)$

A technical issue

$*stream?(x : \mathit{int}, y : t).(\nu a : s)(y!\langle x, a \rangle \mid stream!\langle x + 1, a \rangle)$

$$t = ![\mathit{int} \times s_1]^0$$

$$s_1 = ?[\mathit{int} \times s_2]^1$$

$$s_2 = ?[\mathit{int} \times s_3]^2$$

$$\vdots$$

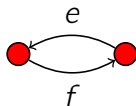
$$s_i = ?[\mathit{int} \times s_{i+1}]^i$$

$$\vdots$$

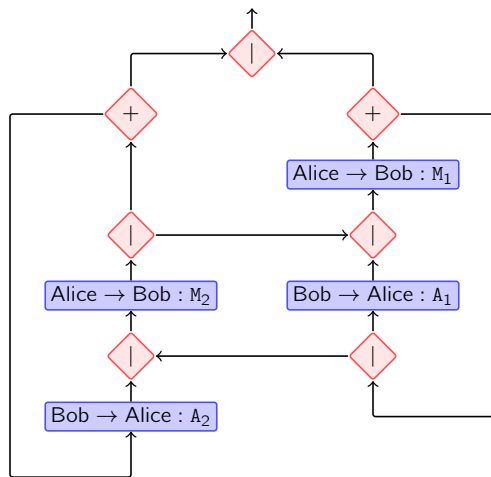
$$t = ![\mathit{int} \times s]^0$$

$$s = ?[\mathit{int} \times s]^1$$

Lock-free full-duplex communication

$$*c?(x^0, y^0).(\nu a^1)(x^0!\langle a^1 \rangle \mid y^0?(z^1).c!\langle a^1, z^1 \rangle)$$
$$c!\langle e, f \rangle \mid c!\langle f, e \rangle$$


Lock-free alternating bit protocol



- Denielou, Yoshida, **Multiparty Session Types Meet Communicating Automata**, ESOP 2012

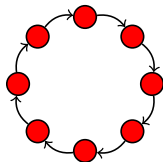
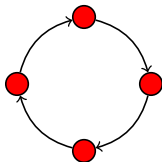
Lock-free alternating bit protocol

$$\text{Bob} \stackrel{\text{def}}{=} *bob?(x^0, y^2). \\ (\nu a^3 b^5)(x^0?(\bar{x}^1).(\bar{x}^1!\langle a^3 \rangle | \\ y^2?(y^3).(\bar{y}^3!\langle b^5 \rangle | bob!\langle a^3, b^5 \rangle)))$$

$$\text{Alice}_1 \stackrel{\text{def}}{=} *alice_1?(x^0, z^1). \\ (\nu a^1 c^2)(x^0!\langle a^1 \rangle | z^1!\langle c^2 \rangle | \\ a^1?(\bar{x}^3).c^2?(\bar{z}^4).alice_1!\langle \bar{x}^3, \bar{z}^4 \rangle)$$

$$\text{Alice}_2 \stackrel{\text{def}}{=} *alice_2?(y^2, z^1). \\ (\nu b^3 c^4)(z^1?(\bar{z}^2).(y^2!\langle b^3 \rangle | \bar{z}^2!\langle c^4 \rangle | \\ b^3?(y^5).alice_2!\langle \bar{y}^5, c^4 \rangle))$$

Lock-free fairy ring

$$*c?(x^0, y^0).(\nu a^1 b^1)(x^0?(z^1).c!\langle z^1, b^1 \rangle \mid c!\langle b^1, a^1 \rangle \mid y^0!\langle a^1 \rangle)$$


Back to sessions

Strategy #1

$$\underline{a?(x).b?(x').a?(y).b?(y').(a!\langle x + x' \rangle \mid b!\langle y + y' \rangle)}$$

⇓

$$\underline{a?(x, a').b?(x', b').a'(y, a'').b'(y', b'').(a''!\langle x + x' \rangle \mid b''!\langle y + y' \rangle)}$$

Strategy #2

$$\begin{array}{c} ?[\text{int}]^m . ![\text{bool}]^n \\ \uparrow \\ ?[\text{int} \times ![\text{bool}]^n]^m \end{array}$$

$$A \xrightarrow{\text{int}} B^m . B \xrightarrow{\text{bool}} C^n$$

Final considerations

- + simple
- + accurate
- unlimited channels (concurrent objects, dining philosophers)

`<troll-mode>`

- (dead)lock freedom $\not\Rightarrow$ sessions

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What's next

Type reconstruction (with Tzu-Chun Chen)

- ① collect constraints
- ② integer programming problem
- ③ solve constraints between types

Unlimited channels?